CHUYÊN ĐỀ : THỰC HIỆN PHÉP TÍNH

DẠNG 1: RÚT GỌN

Bài 1: Thực hiện phép tính:

a, $\frac{2^{12}.3^5-4^6.9^2}{(2^2.3)^6}-\frac{5^{10}.7^3-25^5.49^2}{(125.7)^3+5^9.14^3}$ b, $\frac{2^{18}.18^7.3^3+3^{15}.2^{15}}{2^{10}.6^{15}+3^{14}.15.4^{13}}$ c, $\frac{4^6.9^5+6^9.120}{8^4.3^{12}-6^{11}}$

HD :

a, Ta có: $\frac{2^{12}.3^5-4^6.9^2}{(2^2.3)^6}-\frac{5^{10}.7^3-25^5.49^2}{(125.7)^3+5^9.14^3}=\frac{2^{12}.3^5-\left(2^2\right)^6.\left(3^2\right)^2}{2^{12}.3^6}-\frac{5^{10}.7^3-\left(5^2\right)^5.\left(7^2\right)^2}{\left(5^3\right)^3.7^3+5^9.2^3.7^3}$

$=\frac{2^{12}.3^5-2^{12}.3^4}{2^{12}.3^6}-\frac{5^{10}.7^3-5^{10}.7^4}{5^9.7^3+5^9.2^3.7^3}=\frac{2^{12}.3^4\left(3-1\right)}{2^{12}.3^6}-\frac{5^{10}.7^3\left(1-7\right)}{5^9.7^3\left(1+8\right)}=\frac{2}{3^2}-\frac{5.6}{9}=\frac{-28}{9}$

b, Ta có: $\frac{2^{18}.18^7.3^3+3^{15}.2^{15}}{2^{10}.6^{15}+3^{14}.15.4^{13}}=\frac{2^{18}.2^7.3^{14}.3^3+3^{15}.2^{15}}{2^{10}.2^{15}.3^{15}+3^{14}.3.5.2^{28}}=\frac{2^{25}.3^{17}+3^{15}.2^{15}}{2^{25}.3^{15}+3^{15}.2^{28}.5}$

$=\frac{2^{15}.3^{15}\left(2^{10}.3^2+1\right)}{2^{25}.3^{15}\left(1+2^3.5\right)}=\frac{\left(2^{10}.3^2+1\right)}{2^{10}41}$

c, Ta có: $\frac{4^6.9^5+6^9.120}{8^4.3^{12}-6^{11}}=\frac{\left(2^2\right)^6.\left(3^2\right)^5+2^9.3^9.2^3.3.5}{\left(2^3\right)^4.3^{12}-2^{11}.3^{11}}=\frac{2^{12}.3^{10}+2^{12}.3^{10}.5}{2^{12}.3^{12}-2^{11}.3^{11}}=\frac{2^{12}.3^{10}\left(1+5\right)}{2^{11}.3^{11}\left(2.3-1\right)}=\frac{2.6}{3.5}=\frac{4}{5}$

Bài 2: Thực hiện phép tính:

a, $\frac{5.4^{15}.9^9-4.3^{20}.8^9}{5.2^9.9^{16}-7.2^{29}.27^6}$ b, $\frac{2^4.5^2.11^2.7}{2^3.5^3.7^2.11}$ c, $\frac{5^{11}.7^{12}+5^{11}.7^{11}}{5^{12}.7^{11}+9.5^{11}.7^{11}}$

HD :

a, Ta có: $\frac{5.4^{15}.9^9-4.3^{20}.8^9}{5.2^{29}.3^{16}-7.2^{29}.27^6}=\frac{5.2^{30}.3^{18}-2^{29}.3^{20}}{5.2^{29}.3^{16}-7.2^{29}.3^{18}}=\frac{2^{29}.3^{18}\left(5.2-3^2\right)}{2^{29}.3^{16}\left(5-7.3^2\right)}=\frac{3^2}{-58}=\frac{-9}{58}$

b, Ta có: $\frac{2^4.5^2.11^2.7}{2^3.5^3.7^2.11}=\frac{2.11}{5.7}=\frac{22}{35}$

c, Ta có: $\frac{5^{11}.7^{12}+5^{11}.7^{11}}{5^{12}.7^{11}+9.5^{11}.7^{11}}=\frac{5^{11}.7^{11}\left(7+1\right)}{5^{11}.7^{11}\left(5+9\right)}=\frac{8}{14}=\frac{4}{7}$

Bài 3: Thực hiện phép tính:

a, $\frac{11.3^{22}.3^7-9^{15}}{(2.3^{14})^2}$ b, $\frac{2^{10}.3^{10}-2^{10}.3^9}{2^9.3^{10}}$ c, $\frac{4^5.9^4-2.6^9}{2^{10}.3^8+6^8.20}$

HD :

a, Ta có: $\frac{11.3^{22}.3^7-9^{15}}{(2.3^{14})^2}=\frac{11.3^{29}-3^{30}}{2^2.3^{28}}=\frac{3^{29}.(11-3)}{2^2.3^{28}}=\frac{3.8}{4}=6$

b, Ta có: $\frac{2^{10}.3^{10}-2^{10}.3^9}{2^9.3^{10}}=\frac{2^{10}.3^9\left(3-1\right)}{2^9.3^{10}}=\frac{2.2}{3}=\frac{4}{3}$

c, Ta có: $\frac{4^5.9^4-2.6^9}{2^{10}.3^8+6^8.20}=\frac{2^{10}.3^8-2^{10}.3^9}{2^{10}.3^8+2^{10}.3^8.5}=\frac{2^{10}.3^8\left(1-3\right)}{2^{10}.3^8\left(1+5\right)}=\frac{-2}{6}=\frac{-1}{3}$

Bài 4: Thực hiện phép tính:

a, $\frac{2^{12}.3^5-4^6.9^2}{(2^2.3)^6+8^4.3^5}-\frac{5^{10}.7^3-25^5.49^2}{(125.7)^3+5^9.14^3}$ b, $\frac{5.4^{15}.9^9-4.3^{20}.8^9}{5.2^9.6^{19}-7.2^{29}.27^6}$ c, $\frac{4^5.9^4-2.6^9}{2^{10}.3^8+6^8.20}$

HD:

a, Ta có : $\frac{2^{12}.3^5-4^6.9^2}{(2^2.3)^6+8^4.3^5}-\frac{5^{10}.7^3-25^5.49^2}{(125.7)^3+5^9.14^3}$

$=\frac{2^{12}.3^5-2^{12}.3^4}{2^{12}.3^6+2^{12}.3^5}-\frac{5^{10}.7^3-5^{10}.7^4}{5^9.7^3+5^9.7^3.2^3}=\frac{2^{12}.3^4(3-1)}{2^{12}.3^5(3+1)}-\frac{5^{10}.7^3(1-7)}{5^9.7^3(1+8)}=\frac{5.(-6)}{9}=\frac{-10}{3}$

b, Ta có : $\frac{5.4^{15}.9^9-4.3^{20}.8^9}{5.2^9.6^{19}-7.2^{29}.27^6}=\frac{5.2^{30}.3^{18}-3^{20}.2^{29}}{5.2^{28}.3^{19}-7.2^{29}.3^{18}}=\frac{2^{29}.3^{18}(5.2-3^2)}{2^{28}.3^{18}(5.3-7.2)}=\frac{2}{1}=2$

c, Ta có : $\frac{4^5.9^4-2.6^9}{2^{10}.3^8+6^8.20}=\frac{2^{10}.3^8-2^{10}.3^9}{2^{10}.3^8+2^{10}.3^8.5}=\frac{2^{10}.3^8(1-3)}{2^{10}.3^8(1+5)}=\frac{-2}{6}=\frac{-1}{3}$

Bài 5: Thực hiện phép tính:

a, $\frac{15.4^{12}.9^7-4.3^{15}.8^8}{19.2^{24}.3^{14}-6.4^{12}.27^5}$ b, $\frac{3^{15}.2^{22}+6^{16}.4^4}{2.9^9.8^7-7.27^5.2^{23}}$ c, $\frac{16^3.3^{10}+120.6^9}{4^6.3^{12}+6^{11}}$

HD :

a, Ta có: $\frac{15.4^{12}.9^7-4.3^{15}.8^8}{19.2^{24}.3^{14}-6.4^{12}.27^5}=\frac{5.2^{24}.3^{15}-2^{26}.3^{15}}{19.2^{24}.3^{14}-2^{25}.3^{16}}=\frac{2^{24}.3^{15}(5-2^2)}{2^{24}.3^{24}(19-2.3^2)}=\frac{3}{1}=3$

b, Ta có : $\frac{3^{15}.2^{22}+6^{16}.4^4}{2.9^9.8^7-7.27^5.2^{23}}=\frac{3^{15}.2^{22}+2^{24}.3^{16}}{2^{22}.3^{18}-7.3^{15}.2^{23}}=\frac{2^{22}.3^{15}(1+2^2.3)}{2^{22}.3^{15}(3^3-7.2)}=\frac{13}{-5}=\frac{-13}{5}$

c, Ta có: $\frac{(2^4)^3.3^{10}+2^3.3.5.(2.3)^9}{(2^2)^6.3^{12}+(2.3)^{11}}=\frac{2^{12}.3^{10}+2^{12}.3^{10}.5}{2^{12}.3^{12}+2^{11}.3^{11}}=\frac{2^{12}.3^{10}(1+5)}{2^{11}.3^{11}(2.3+1)}=\frac{2.6}{3.7}=\frac{12}{21}$

Bài 6: Thực hiện phép tính :

a, $A=\frac{2^{12}.3^5-4^6.9^2}{(2^2.3)^6+8^4.3^5}-\frac{5^{10}.7^3-25^5.49^2}{(125.7)^3+5^9.14^3}$ b, $\frac{5.4^{15}.9^9-4.3^{20}.8^9}{5.2^{10}.6^{12}-7.2^{29}.27^6}$

Bài 7: Thực hiện phép tính:

a, $A=\frac{2^{12}.3^5-4^6.9^2}{(2^2.3)^6+8^4.3^5}$ b, $B=\frac{4^5.9^4-2.6^9}{2^{10}.3^8+6^8.20}$

Bài 8: Thực hiện phép tính :

a, $\frac{3^{10}.11+3^{10}.5}{3^9.2^4}$ b, $\frac{2^{10}.13+2^{10}.65}{2^8.104}$

Bài 9: Thực hiện phép tính:

a, $\frac{2^{30}.5^7+2^{13}.5^{27}}{2^{27}.5^7+2^{10}.5^{27}}$ b, $\frac{(-3)^6.15^5+9^3.(-15)^6}{(-3)^{10}.5^5.2^3}$

Bài 10: Thực hiện phép tính:

a, $\frac{5^2.6^{11}.16^2+6^2.12^6.15^2}{2.6^{12}.10^4-81^2.960^3}$ b, $A=\frac{2^{19}.27^3.5-15.(-4)^9.9^4}{6^9.2^{10}-(-12)^{10}}$

Bài 11: Thực hiện phép tính:

a, $\left[\frac{(0,8)^5}{(0,4)^6}+\frac{2^{15}.9^4}{6^6.8^3}\right]:\frac{45^{10}.5^{20}}{75^{15}}$ b, $A=\frac{2.5^{22}-9.5^{21}}{25^{10}}:\frac{5(3.7^{15}-19.7^{14})}{7^{16}+3.7^{15}}$

Bài 12: Tính giá trị của biểu thức: $A=\frac{\left(\frac{2}{5}\right)^7.5^7+\left(\frac{9}{4}\right)^3:\left(\frac{3}{16}\right)^3}{2^7.5^7+512}$

Bài 13: Tính biểu thức: $B=\sqrt{2\frac{14}{25}}-\sqrt{1,21}+\frac{0,6-\frac{3}{7}-\frac{3}{13}}{1,2-\frac{6}{7}-\frac{6}{13}}:\frac{-1\frac{1}{6}+0,875-0,7}{\frac{1}{3}-0,25+0,2}$

Bài 14: Tính biểu thức: $A=-84\left(\frac{-1}{3}+\frac{1}{4}-\frac{1}{7}\right)+51.(-37)-51.(-137)+\frac{3^3.12^6}{\left(27.4^2\right)^3}$

Bài 15: Thực hiện phép tính:

a, 1024: $(17.2^5+15.2^5)$ b, $5^3.2+(23+4^0):2^3$ c, $(5.3^5+17.3^4):6^2$

HD :

a, Ta có: 1024: $(17.2^5+15.2^5)=2^{10}:\left[2^5(17+15)\right]=2^{10}:\left(2^5.2^5\right)=1$

b, Ta có: $5^3.2+(23+4^0):2^3=5^3.2+24:2^3=250+3=253$

c, Ta có: $(5.3^5+17.3^4):6^2\ \left[3^4(3.5+17)\right]:3^2.2^2=\left(3^4.32\right):3^2.2^2=\frac{3^4.2^5}{3^2.2^2}=9.8=72$

Bài 16: Thực hiện phép tính:

a, $(10^2+11^2+12^2):(13^2+14^2)$ b, $(2^3.9^4+9^3.45):(9^2.10-9^2)$

HD :

a, Ta có: $(10^2+11^2+12^2):(13^2+14^2)=(100+121+144):(169+196)=365:365=1$

c Ta có : $(2^3.9^4+9^3.45):(9^2.10-9^2)=\left(2^3.3^8+3^{11}.5\right):\left(3^2.10+3^2\right)=\frac{3^8\left(8+3^3.5\right)}{3^2.11}=\frac{3^6.143}{11}=13.3^6$

Bài 17: Thực hiện phép tính:

a, $\left[(3^{14}.69+3^{14}.12):3^{16}-7\right]:2^4$ b, $24^4:3^4-32^{12}:16^{12}$

HD :

a, Ta có:

$\left[(3^{14}.69+3^{14}.12):3^{16}-7\right]:2^4=\left[\left(3^{14}.3.23+3^{14}.3.2^2\right):3^{16}-7\right]:2^4=\left[\left(3^{15}.23+3^{15}.4\right):3^{16}-7\right]:2^4$

$=\left[3^{15}.27:3^{16}-7\right]:2^4=(9-7):2^4=\frac{1}{2^3}$

b, Ta có: $24^4:3^4-32^{12}:16^{12}=(24:3)^4-(32:16)^{12}=8^4-2^{12}=2^{12}-2^{12}=0$

Bài 18: Thực hiện phép tính :

a, $2010^{2010}\left(7^{10}:7^8-3.2^4-2^{2010}:2^{2010}\right)$ b, $\left(2^{100}+2^{101}+2^{102}\right):\left(2^{97}+2^{98}+2^{99}\right)$

HD :

a, Ta có : $2010^{2010}\left(7^{10}:7^8-3.2^4-2^{2010}:2^{2010}\right)=2010^{2010}(49-3.16-1)=0$

Bài 19: Tính: $A=\frac{\frac{-11}{2}+\frac{\frac{-5}{3}}{1-\frac{4}{3}}}{\frac{3}{5}-\frac{\frac{-2}{5}}{\frac{4}{5}-\frac{2}{3}}}$ $B=\frac{1-\frac{1}{1+\frac{4}{3}}}{2+\frac{1}{3}-\frac{3}{7}}$

Bài 20: Thực hiện phép tính : $\frac{45}{19}-\left(\frac{1}{2}+\left(\frac{1}{3}+\left(\frac{1}{4}\right)^{-1}\right)^{-1}\right)^{-1}$

HD :

$$= \frac{45}{19} - \frac{1}{\frac{1}{2} + \frac{1}{\frac{1}{3} + 4}} = \frac{45}{19} - \frac{26}{19} = 1$$

Bài 21: Rút gọn : $A = \left(\frac{3}{2} - \frac{2}{5} + \frac{1}{10}\right) : \left(\frac{3}{2} - \frac{2}{3} + \frac{1}{12}\right)$

DẠNG 2 : TÍNH ĐƠN GIẢN

Bài 1: Thực hiện phép tính: $\dfrac{\dfrac{1}{2003}+\dfrac{1}{2004}-\dfrac{1}{2005}}{\dfrac{5}{2003}+\dfrac{5}{2004}-\dfrac{5}{2005}}-\dfrac{\dfrac{2}{2002}+\dfrac{2}{2003}-\dfrac{2}{2004}}{\dfrac{3}{2002}+\dfrac{3}{2003}-\dfrac{3}{2004}}$

HD:

Ta có : $\dfrac{\dfrac{1}{2003}+\dfrac{1}{2004}-\dfrac{1}{2005}}{\dfrac{5}{2003}+\dfrac{5}{2004}-\dfrac{5}{2005}}-\dfrac{\dfrac{2}{2002}+\dfrac{2}{2003}-\dfrac{2}{2004}}{\dfrac{3}{2002}+\dfrac{3}{2003}-\dfrac{3}{2004}}=$

$\dfrac{\dfrac{1}{2003}+\dfrac{1}{2004}-\dfrac{1}{2005}}{5\left(\dfrac{1}{2003}+\dfrac{1}{2004}-\dfrac{1}{2005}\right)}-\dfrac{2\left(\dfrac{1}{2002}+\dfrac{1}{2003}-\dfrac{1}{2004}\right)}{3\left(\dfrac{1}{2002}+\dfrac{1}{2003}-\dfrac{1}{2004}\right)}=\dfrac{1}{5}-\dfrac{2}{3}=\dfrac{-7}{15}$

Bài 2: Thực hiện phép tính: $\left(\dfrac{1,5+1-0,75}{2,5+\dfrac{5}{3}-1.25}+\dfrac{0,375-0,3+\dfrac{3}{11}+\dfrac{3}{12}}{-0,625+0,5-\dfrac{5}{11}-\dfrac{5}{12}}\right):\dfrac{1890}{2005}+115$

HD:

Ta có : $\left(\dfrac{1,5+1-0,75}{2,5+\dfrac{5}{3}-1.25}+\dfrac{0,375-0,3+\dfrac{3}{11}+\dfrac{3}{12}}{-0,625+0,5-\dfrac{5}{11}-\dfrac{5}{12}}\right):\dfrac{1890}{2005}+115$

$=\left(\dfrac{\dfrac{3}{2}+\dfrac{3}{3}-\dfrac{3}{4}}{\dfrac{5}{2}+\dfrac{5}{3}-\dfrac{5}{4}}+\dfrac{\dfrac{3}{8}-\dfrac{3}{10}+\dfrac{3}{11}+\dfrac{3}{12}}{\dfrac{-5}{8}+\dfrac{5}{10}-\dfrac{5}{11}-\dfrac{5}{12}}\right):\dfrac{378}{401}+115=\left(\dfrac{3}{5}+\dfrac{3}{-5}\right):\dfrac{378}{401}+115=0:\dfrac{378}{401}+115=115$

Bài 3: Thực hiện phép tính: $\dfrac{\dfrac{1}{9}-\dfrac{1}{7}-\dfrac{1}{11}}{\dfrac{4}{9}-\dfrac{4}{7}-\dfrac{4}{11}}+\dfrac{0,6-\dfrac{3}{25}-\dfrac{3}{125}-\dfrac{3}{625}}{\dfrac{4}{5}-0,16-\dfrac{4}{125}-\dfrac{4}{625}}$

HD:

Ta có : $\dfrac{\dfrac{1}{9}-\dfrac{1}{7}-\dfrac{1}{11}}{\dfrac{4}{9}-\dfrac{4}{7}-\dfrac{4}{11}}+\dfrac{0,6-\dfrac{3}{25}-\dfrac{3}{125}-\dfrac{3}{625}}{\dfrac{4}{5}-0,16-\dfrac{4}{125}-\dfrac{4}{625}}=\dfrac{1}{4}+\dfrac{3}{4}=1$

Bài 4: Thực hiện phép tính: $564.\left(\dfrac{12+\dfrac{12}{7}-\dfrac{12}{25}-\dfrac{12}{71}}{4+\dfrac{4}{7}-\dfrac{4}{25}-\dfrac{4}{71}}:\dfrac{3+\dfrac{3}{13}+\dfrac{3}{19}+\dfrac{3}{101}}{5+\dfrac{5}{13}+\dfrac{5}{19}+\dfrac{5}{101}}\right)$

HD:

Ta có : $564.\left(\dfrac{12+\dfrac{12}{7}-\dfrac{12}{25}-\dfrac{12}{71}}{4+\dfrac{4}{7}-\dfrac{4}{25}-\dfrac{4}{71}}:\dfrac{3+\dfrac{3}{13}+\dfrac{3}{19}+\dfrac{3}{101}}{5+\dfrac{5}{13}+\dfrac{5}{19}+\dfrac{5}{101}}\right)=564.\left(\dfrac{12}{4}:\dfrac{3}{5}\right)=564.5=2820$

Bài 5: Thực hiện phép tính:

a, $\dfrac{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}}{1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}}$ b, $\dfrac{5-\frac{5}{3}+\frac{5}{9}-\frac{5}{27}}{8-\frac{8}{3}+\frac{8}{9}-\frac{8}{27}}:\dfrac{15-\frac{15}{11}+\frac{15}{121}}{16-\frac{16}{11}+\frac{16}{121}}$

HD:

a, Ta có : $\dfrac{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}}{1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}}=\dfrac{16\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)}{16\left(1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}\right)}=\dfrac{16+8+4+2+1}{16-8+4-2+1}=\dfrac{31}{11}$

b, Ta có : $\dfrac{5-\frac{5}{3}+\frac{5}{9}-\frac{5}{27}}{8-\frac{8}{3}+\frac{8}{9}-\frac{8}{27}}:\dfrac{15-\frac{15}{11}+\frac{15}{121}}{16-\frac{16}{11}+\frac{16}{121}}=\dfrac{5}{8}:\dfrac{15}{16}=\dfrac{5}{8}.\dfrac{16}{15}=\dfrac{2}{3}$

Bài 6: Thực hiện phép tính:

a, $\dfrac{2-\frac{2}{19}+\frac{2}{43}-\frac{2}{1943}}{3-\frac{3}{19}+\frac{3}{43}-\frac{3}{1943}}:\dfrac{4-\frac{4}{29}+\frac{4}{41}-\frac{4}{2941}}{5-\frac{5}{29}+\frac{5}{41}-\frac{5}{2941}}$ b, $\dfrac{12-\frac{12}{7}-\frac{12}{289}-\frac{12}{85}}{4-\frac{4}{7}-\frac{4}{289}-\frac{4}{85}}:\dfrac{3+\frac{3}{13}+\frac{3}{169}+\frac{3}{91}}{7+\frac{7}{13}+\frac{7}{169}+\frac{7}{91}}$

HD:

a, Ta có : $\dfrac{2-\frac{2}{19}+\frac{2}{43}-\frac{2}{1943}}{3-\frac{3}{19}+\frac{3}{43}-\frac{3}{1943}}:\dfrac{4-\frac{4}{29}+\frac{4}{41}-\frac{4}{2941}}{5-\frac{5}{29}+\frac{5}{41}-\frac{5}{2941}}=\dfrac{2}{3}:\dfrac{4}{5}=\dfrac{2}{3}.\dfrac{5}{4}=\dfrac{5}{6}$

b, Ta có : $\dfrac{12-\frac{12}{7}-\frac{12}{289}-\frac{12}{85}}{4-\frac{4}{7}-\frac{4}{289}-\frac{4}{85}}:\dfrac{3+\frac{3}{13}+\frac{3}{169}+\frac{3}{91}}{7+\frac{7}{13}+\frac{7}{169}+\frac{7}{91}}=\dfrac{12}{4}:\dfrac{3}{7}=3.\dfrac{7}{3}=7$

Bài 7: Thực hiện phép tính:

a, $\dfrac{\left(\frac{-5}{7}-\frac{7}{9}+\frac{9}{11}-\frac{11}{13}\right)\left(3-\frac{3}{4}\right)}{\left(\frac{10}{21}+\frac{14}{27}-\frac{6}{11}+\frac{22}{39}\right):\left(2-\frac{2}{3}\right)}$ b, $\dfrac{3+\frac{3}{7}-\frac{3}{11}+\frac{3}{1001}-\frac{3}{13}}{\frac{9}{1001}-\frac{9}{13}+\frac{9}{7}-\frac{9}{11}+9}$

HD:

a, Ta có : $\dfrac{\left(\frac{-5}{7}-\frac{7}{9}+\frac{9}{11}-\frac{11}{13}\right)\left(3-\frac{3}{4}\right)}{\left(\frac{10}{21}+\frac{14}{27}-\frac{6}{11}+\frac{22}{39}\right):\left(2-\frac{2}{3}\right)}=\dfrac{-\left(\frac{5}{7}+\frac{7}{9}-\frac{9}{11}+\frac{11}{13}\right).\frac{9}{4}}{\frac{2}{3}\left(\frac{5}{7}+\frac{7}{9}-\frac{9}{11}+\frac{11}{13}\right):\frac{4}{3}}=\dfrac{\frac{-9}{4}}{\frac{2}{3}.\frac{3}{4}}=\dfrac{-9}{4}:\dfrac{1}{2}=\dfrac{-9}{2}$

b, Ta có : $\dfrac{3+\frac{3}{7}-\frac{3}{11}+\frac{3}{1001}-\frac{3}{13}}{\frac{9}{1001}-\frac{9}{13}+\frac{9}{7}-\frac{9}{11}+9}=\dfrac{3\left(1+\frac{1}{7}-\frac{1}{11}+\frac{1}{1001}-\frac{1}{13}\right)}{9\left(1+\frac{1}{7}-\frac{1}{11}+\frac{1}{1001}-\frac{1}{13}\right)}=\dfrac{3}{9}=\dfrac{1}{3}$

Bài 8: Tính nhanh: $\dfrac{50-\frac{4}{13}+\frac{2}{15}-\frac{2}{17}}{100-\frac{8}{13}+\frac{4}{15}-\frac{4}{17}}$

HD:

Ta có : $\dfrac{50-\dfrac{4}{13}+\dfrac{2}{15}-\dfrac{2}{17}}{100-\dfrac{8}{13}+\dfrac{4}{15}-\dfrac{4}{17}}=\dfrac{50-\dfrac{4}{13}+\dfrac{2}{15}-\dfrac{2}{17}}{2\left(50-\dfrac{4}{13}+\dfrac{4}{15}-\dfrac{4}{17}\right)}=\dfrac{1}{2}$

Bài 9: Tính:

a, A= $\dfrac{24.47-23}{24+47.23}.\dfrac{3+\dfrac{3}{7}-\dfrac{3}{11}+\dfrac{3}{1001}-\dfrac{3}{13}}{\dfrac{9}{1001}-\dfrac{9}{13}+\dfrac{9}{7}-\dfrac{9}{11}+9}$

b, $\dfrac{\dfrac{2}{3}+3.\dfrac{2}{3}-\left(\dfrac{5}{6}\right)^2}{\dfrac{7}{60}:\left(\dfrac{35}{31.37}+\dfrac{35}{37.43}+\dfrac{105}{43.61}+\dfrac{35}{61.67}\right)}$

HD:

a, Ta có : $\dfrac{24.47-23}{24+47.23}=\dfrac{47\left(23+1\right)-23}{47.23+24}=\dfrac{47.23+24}{47.23+24}=1$

và $\dfrac{3\left(1+\dfrac{1}{7}-\dfrac{1}{11}+\dfrac{1}{1001}-\dfrac{1}{13}\right)}{9\left(1+\dfrac{1}{7}-\dfrac{1}{11}+\dfrac{1}{1001}-\dfrac{1}{13}\right)}=\dfrac{3}{9}=>A=\dfrac{3}{9}=\dfrac{1}{3}$

b, Ta có :

$TS=\dfrac{2}{3}+3.\dfrac{2}{3}-\dfrac{25}{36}=\dfrac{8}{3}-\dfrac{25}{36}=\dfrac{71}{36}$

$MS=\dfrac{7}{60}:\left(\dfrac{5.7}{31.37}+\dfrac{5.7}{37.43}+\dfrac{3.5.7}{43.61}+\dfrac{5.7}{61.67}\right)=\dfrac{7}{6}:\left[\dfrac{35}{6}\left(\dfrac{6}{31.37}+\dfrac{6}{37.43}+\dfrac{18}{43.61}+\dfrac{6}{61.67}\right)\right]$

$MS=\dfrac{7}{60}:\left[\dfrac{35}{6}\left(\dfrac{1}{31}-\dfrac{1}{37}+\dfrac{1}{37}-\dfrac{1}{43}+\dfrac{1}{43}-\dfrac{1}{61}+\dfrac{1}{61}-\dfrac{1}{67}\right)\right]$

$MS=\dfrac{7}{60}:\left[\dfrac{35}{6}\left(\dfrac{1}{31}-\dfrac{1}{67}\right)\right]=\dfrac{2077}{1800}=>B=\dfrac{71}{36}:\dfrac{2077}{1800}$

Câu 10: Thực hiện phép tính:

a, $A=\dfrac{155-\dfrac{10}{7}-\dfrac{5}{11}+\dfrac{5}{23}}{402-\dfrac{26}{7}-\dfrac{13}{11}+\dfrac{13}{23}}+\dfrac{\dfrac{3}{5}+\dfrac{3}{13}-0,9}{\dfrac{7}{91}+0,2-\dfrac{3}{10}}$

b, $A=\dfrac{0,375-0,3+\dfrac{3}{11}+\dfrac{3}{12}}{-0,625+0,5-\dfrac{5}{11}-\dfrac{5}{12}}+\dfrac{1,5+1-0,75}{2,5+\dfrac{5}{3}-1,25}$

DẠNG 3 : TÍNH TỔNG TỰ NHIÊN

Bài 1: Tính tổng tự nhiên

a, A= $9+99+999+...+999...9$ (10 số 9) b, B= $1+11+111+...+111...1$ (10 số 1)

HD:

a, Ta có: $A=(10-1)+(10^2-1)+(10^3-1)+...+(10^{10}-1)$

$=(10+10^2+10^3+...+10^{10})-10=111...10-10=111...100$ (9 số 1)

b, Ta có: $9B=9+99+999+....+9999...99$ (10 số 9). Tính như câu a

Bài 2: Tính tổng tự nhiên

a, C= $4+44+444+...+444...4$ (10 số 4) b, D= $2+22+222+...+222...2$ (10 số 2)

HD:

a, Ta có: $C=4(1+11+111+...+111...11)$ (10 số 1)

$9C=4(9+99+999+...+999...99)$ (10 số 9). Tính như tính ở trên

b, Ta có :

$D=2(1+11+111+...+111...11)$ (10 số 1)

$9D=2(9+99+999+...+999...99)$ (10 số 9)

Bài 3 : Tính tổng sau: E= $3+33+333+...+333...3$ (10 số 3)

DẠNG 4 : TÍNH TỔNG PHÂN SỐ

Bài 1: Tính nhanh tổng sau:

a, A= $\frac{1}{5.6}+\frac{1}{6.7}+...+\frac{1}{24.25}$ b, B= $\frac{2}{1.3}+\frac{2}{3.5}+\frac{2}{5.7}+...+\frac{2}{99.101}$

HD:

a, Ta có : $A=\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+...+\left(\frac{1}{24}-\frac{1}{25}\right)=\frac{1}{5}-\frac{1}{25}=\frac{4}{25}$

b, Ta có : $B=\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+...+\left(\frac{1}{99}-\frac{1}{101}\right)=1-\frac{1}{101}=\frac{100}{101}$

Bài 2: Tính nhanh tổng sau:

a, D= $\frac{5^2}{1.6}+\frac{5^2}{6.11}+...+\frac{5^2}{26.31}$ b, K= $\frac{4}{11.16}+\frac{4}{16.21}+\frac{4}{21.26}+...+\frac{4}{61.66}$

HD :

a, Ta có : $D=5\left(\frac{5}{1.6}+\frac{5}{6.11}+\frac{5}{11.16}+....+\frac{5}{26.31}\right)=5\left(1-\frac{1}{6}+\frac{1}{6}-\frac{1}{11}+\frac{1}{11}-\frac{1}{16}+...+\frac{1}{26}-\frac{1}{31}\right)$

$D=5\left(1-\frac{1}{31}\right)=5.\frac{30}{31}=\frac{150}{31}$

b, Ta có: $K=4\left(\frac{1}{11.16}+\frac{1}{16.21}+\frac{1}{21.26}+...+\frac{1}{61.66}\right)\Rightarrow 5K=4\left(\frac{5}{11.16}+\frac{5}{16.21}+\frac{5}{21.26}+...+\frac{5}{61.66}\right)$

$5K=4\left(\frac{1}{11}-\frac{1}{16}+\frac{1}{16}-\frac{1}{21}+...+\frac{1}{61}-\frac{1}{66}\right)=4\left(\frac{1}{11}-\frac{1}{66}\right)\Rightarrow 5K=4.\frac{55}{11.66}\Rightarrow K=\frac{4}{66}=\frac{2}{33}$

Bài 3: Tính nhanh tổng sau:

a, N= $\frac{4}{1.3}+\frac{4}{3.5}+\frac{4}{5.7}+...+\frac{4}{99.101}$ b, $F=\frac{1}{1.1985}+\frac{1}{2.1986}+\frac{1}{3.1987}+...+\frac{1}{16.2000}$

HD :

a, Ta có : $N=2\left(\frac{2}{1.3}+\frac{2}{3.5}+\frac{2}{5.7}+...+\frac{2}{99.101}\right)=2\left(1-\frac{1}{101}\right)=\frac{200}{101}$

Bài 4: Tính tổng sau: $K=\frac{5}{3.7}+\frac{5}{7.11}+\frac{5}{11.15}+...+\frac{5}{81.85}+\frac{5}{85.89}$

Bài 5: Tính tổng sau: $A=\frac{1}{25.24}+\frac{1}{24.23}+...+\frac{1}{7.6}+\frac{1}{6.5}$

Bài 6:Tính tổng sau: $A=\frac{5}{3.6}+\frac{5}{6.9}+\frac{5}{9.12}+...+\frac{5}{99.102}$

Bài 7: Tính giá trị của biểu thức:

$A=\left(\frac{3}{1.8}+\frac{3}{8.15}+\frac{3}{15.22}+...+\frac{3}{106.113}\right)-\left(\frac{25}{50.55}+\frac{25}{55.60}+...+\frac{25}{95.100}\right)$

HD:

Ta có : $B=\frac{3}{1.8}+\frac{3}{8.15}+\frac{3}{15.22}+...+\frac{3}{106.113}\Rightarrow 7B=3\left(\frac{7}{1.8}+\frac{7}{8.15}+\frac{7}{15.22}+...+\frac{7}{106.113}\right)$

$\Rightarrow 7B=3\left(\frac{1}{1}-\frac{1}{8}+\frac{1}{8}-\frac{1}{15}+\frac{1}{15}-\frac{1}{22}+...+\frac{1}{106}-\frac{1}{113}\right)=3\left(1-\frac{1}{113}\right)=3.\frac{112}{113}\Rightarrow B=\frac{3.112}{7.113}=\frac{48}{113}$

và $C=\frac{25}{50.55}+\frac{25}{55.60}+...+\frac{25}{95.100}\Rightarrow 5C=\frac{5}{50.55}+\frac{5}{55.60}+...+\frac{5}{95.100}$

$\Rightarrow 5C=\frac{1}{50}-\frac{1}{100}=\frac{1}{100}\Rightarrow C=\frac{1}{500}$. Khi đó : $A=B-C=\frac{48}{113}-\frac{1}{500}$

Bài 8: Tính nhanh: $\frac{1}{19}+\frac{9}{19.29}+\frac{9}{29.39}+...+\frac{9}{1999.2009}$

HD:

Ta có : $\frac{1}{19}+\frac{9}{19.29}+\frac{9}{29.39}+...+\frac{9}{1999.2009} \Rightarrow A=\frac{9}{9.19}+\frac{9}{19.29}+\frac{9}{29.39}+...+\frac{9}{1999.2009}$

$\Rightarrow 10A=9\left(\frac{10}{9.19}+\frac{10}{19.29}+\frac{10}{29.39}+...+\frac{10}{1999.2009}\right)=9\left(\frac{1}{9}-\frac{1}{2009}\right)$

$10A=9.\frac{2000}{9.2009}=\frac{2000}{2009}\Rightarrow A=\frac{200}{2009}$

Bài 9: Tính tổng sau: C= $\frac{2}{15}+\frac{2}{35}+\frac{2}{63}+\frac{2}{99}+\frac{2}{143}$

Bài 10: Tính nhanh tổng sau:

a, E= $\frac{1}{7}+\frac{1}{91}+\frac{1}{247}+\frac{1}{475}+\frac{1}{755}+\frac{1}{1147}$ b, C= $\frac{2}{15}+\frac{2}{35}+\frac{2}{63}+\frac{2}{99}+\frac{2}{143}$

HD:

a, Ta có : $E=\frac{1}{1.7}+\frac{1}{7.13}+\frac{1}{13.19}+...+\frac{1}{31.37}=1-\frac{1}{7}+\frac{1}{7}-\frac{1}{13}+...+\frac{1}{31}-\frac{1}{37}=1-\frac{1}{37}=\frac{36}{37}$

b, Ta có : $C=\frac{2}{3.5}+\frac{2}{5.7}+\frac{2}{7.9}+\frac{2}{9.11}+\frac{2}{11.13}=\frac{1}{3}-\frac{1}{11}=\frac{8}{33}$

Bài 11: Tính nhanh tổng sau

a, F= $\frac{2}{6}+\frac{2}{66}+\frac{2}{176}+...+\frac{2}{(5n-4)(5n+1)}$ b, G= $1+\frac{9}{45}+\frac{9}{105}+\frac{9}{189}+...+\frac{9}{29997}$

HD:

a, Ta có : $F=2\left(\frac{1}{6}+\frac{1}{66}+\frac{1}{176}+...+\frac{1}{(5n-4)(5n+1)}\right)=2\left(\frac{1}{1.6}+\frac{1}{6.11}+\frac{1}{11.16}+\frac{1}{(5n-4)(5n+1)}\right)$

$5F=2\left(\frac{5}{1.6}+\frac{5}{6.11}+\frac{5}{11.16}+...+\frac{5}{(5n-4)(5n+1)}\right)=2\left(1-\frac{1}{5n+1}\right)=2.\frac{5n}{5n+1}\Rightarrow F=\frac{2n}{5n+1}$

b, Ta có : $G=1+\frac{3}{15}+\frac{3}{35}+\frac{3}{63}+...+\frac{3}{9999}=1+\frac{3}{3.5}+\frac{3}{5.7}+\frac{3}{7.9}+...+\frac{3}{99.101}$

$G=1+3\left(\frac{1}{3.5}+\frac{1}{5.7}+...+\frac{1}{99.101}\right)\Rightarrow 2G=2+3\left(\frac{2}{3.5}+\frac{2}{5.7}+...+\frac{2}{99.101}\right)$

$2G=2+3\left(\frac{1}{3}-\frac{1}{101}\right)=2+3.\frac{98}{3.101}=2+\frac{98}{101}=\frac{300}{101}\Rightarrow G=\frac{150}{101}$

Bài 12: Tính nhanh tổng sau : M= $\frac{1}{2.15}+\frac{1}{15.3}+\frac{1}{3.21}+...+\frac{6}{87.90}$

HD:

Ta có : $M=\frac{6}{12.15}+\frac{6}{15.18}+\frac{6}{18.21}+...+\frac{6}{87.90}$

$M=2\left(\frac{3}{12.15}+\frac{3}{15.18}+...+\frac{3}{87.90}\right)=2\left(\frac{1}{12}-\frac{1}{15}+\frac{1}{15}-\frac{1}{18}+...+\frac{1}{87}-\frac{1}{90}\right)=2\left(\frac{1}{12}-\frac{1}{90}\right)$

Bài 13 : Tính : $50+\frac{50}{3}+\frac{25}{3}+\frac{20}{4}+\frac{10}{3}+\frac{100}{6.7}+...+\frac{100}{98.99}+\frac{1}{99}$

Bài 14: Tính: $1+\frac{3}{15}+\frac{3}{35}+\frac{3}{63}+...+\frac{3}{99.101}$

Bài 15: Tính: $A=\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+...+\frac{1}{90}+\frac{1}{110}$

Bài 16: Tính: $\frac{1}{2}+\frac{1}{14}+\frac{1}{35}+\frac{1}{65}+\frac{1}{104}+\frac{1}{152}$

Bài 17: Tính tổng: $B=\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\frac{1}{28}+\frac{1}{36}+\frac{1}{45}+\frac{1}{55}+\frac{1}{66}$

Bài 18: Tính nhanh tổng sau

a, H= $\frac{38}{25}+\frac{9}{10}-\frac{11}{15}+\frac{13}{21}-\frac{15}{28}+\frac{17}{36}-...+\frac{197}{4851}-\frac{199}{4950}$ b, I= $\frac{3}{1.2}-\frac{5}{2.3}+\frac{7}{3.4}-...-\frac{201}{100.101}$

HD:

a, Ta có : $\frac{H}{2}=\frac{38}{50}+\frac{9}{20}-\frac{11}{30}+\frac{13}{42}-\frac{15}{56}+...+\frac{197}{9702}-\frac{199}{9900}$

$\frac{H}{2}=\frac{38}{50}+\frac{9}{4.5}-\frac{11}{5.6}+\frac{13}{6.7}-\frac{15}{7.8}+...+\frac{197}{98.99}-\frac{199}{99.100}$

$\frac{H}{2}=\frac{38}{50}+\left(\frac{1}{4}+\frac{1}{5}\right)-\left(\frac{1}{5}+\frac{1}{6}\right)+\left(\frac{1}{6}+\frac{1}{7}\right)-\left(\frac{1}{7}+\frac{1}{8}\right)+...+\left(\frac{1}{98}+\frac{1}{99}\right)-\left(\frac{1}{99}+\frac{1}{100}\right)$

$\frac{H}{2}=\frac{38}{50}+\frac{1}{4}-\frac{1}{100}=\frac{76+25-1}{100}=1\Rightarrow H=2$

b, Ta có : $I=\left(1+\frac{1}{2}\right)-\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{4}\right)-...-\left(\frac{1}{100}+\frac{1}{101}\right)=1-\frac{1}{101}=\frac{100}{101}$

Bài 19: Thực hiện phép tính: $A=3.\frac{1}{1.2}-5.\frac{1}{2.3}+7.\frac{1}{3.4}-...+15.\frac{1}{7.8}-17.\frac{1}{8.9}$

HD:

Ta có : $A=3.\frac{1}{1.2}-5.\frac{1}{2.3}+7.\frac{1}{3.4}-...+15.\frac{1}{7.8}-17.\frac{1}{8.9}=\frac{3}{1.2}-\frac{5}{2.3}+\frac{7}{3.4}-...+\frac{15}{7.8}-\frac{17}{8.9}$

$=\left(\frac{1}{1}+\frac{1}{2}\right)-\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{4}\right)-...+\left(\frac{1}{7}+\frac{1}{8}\right)-\left(\frac{1}{8}+\frac{1}{9}\right)=1-\frac{1}{9}=\frac{8}{9}$

Bài 20: Không quy đồng, Hãy tính : $B=\frac{5}{2.1}+\frac{4}{1.11}+\frac{3}{11.2}+\frac{1}{2.15}+\frac{13}{15.4}$

HD:

Ta có : $B=\frac{5}{2.1}+\frac{4}{1.11}+\frac{3}{11.2}+\frac{1}{2.15}+\frac{13}{15.4}\Rightarrow\frac{B}{7}=\frac{5}{2.7}+\frac{4}{7.11}+\frac{3}{11.14}+\frac{1}{14.15}+\frac{13}{15.28}$

$\Rightarrow\frac{B}{7}=\frac{1}{2}-\frac{1}{7}+\frac{1}{7}-\frac{1}{11}+\frac{1}{11}-\frac{1}{14}+\frac{1}{14}-\frac{1}{15}+\frac{1}{15}-\frac{1}{28}=\frac{1}{2}-\frac{1}{28}=\frac{13}{28}\Rightarrow B=\frac{7.13}{28}=\frac{13}{4}$

Bài 21: Tính : $A=\frac{4}{7.31}+\frac{6}{7.41}+\frac{9}{10.41}+\frac{7}{10.57}$ và $B=\frac{7}{19.31}+\frac{5}{19.43}+\frac{3}{23.43}+\frac{11}{23.57}$

HD:

Ta có :

$\frac{A}{5}=\frac{4}{31.35}+\frac{6}{35.41}+\frac{9}{41.50}+\frac{7}{50.57}=\left(\frac{1}{31}-\frac{1}{35}\right)+\left(\frac{1}{35}-\frac{1}{41}\right)+\left(\frac{1}{41}-\frac{1}{50}\right)+\left(\frac{1}{50}-\frac{1}{57}\right)=\frac{1}{31}-\frac{1}{57}$

$\frac{B}{2}=\frac{7}{31.38}+\frac{5}{38.43}+\frac{3}{43.46}+\frac{11}{46.57}=\left(\frac{1}{31}-\frac{1}{38}\right)+\left(\frac{1}{38}-\frac{1}{43}\right)+\left(\frac{1}{43}-\frac{1}{46}\right)+\left(\frac{1}{46}-\frac{1}{57}\right)=\frac{1}{31}-\frac{1}{57}$

Khi đó : $\frac{A}{5}=\frac{B}{2}\Rightarrow\frac{A}{B}=\frac{5}{2}$

Bài 22: Tính nhanh tổng sau: P= $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...+\frac{1}{10.11.12}$

HD:

Ta có : $2P=\frac{2}{1.2.3}+\frac{2}{2.3.4}+\frac{2}{3.4.5}+...+\frac{2}{10.11.12}=\left(\frac{1}{1.2}-\frac{1}{2.3}\right)+\left(\frac{1}{2.3}-\frac{1}{3.4}\right)+...+\left(\frac{1}{10.11}-\frac{1}{11.12}\right)$

$$2P=\frac{1}{1.2}-\frac{1}{11.12}=\frac{65}{132}\Rightarrow P=\frac{65}{264}$$

Bài 23 : Tính : $A=\frac{1}{10}+\frac{1}{40}+\frac{1}{88}+\frac{1}{154}+\frac{1}{238}+\frac{1}{340}$

Bài 24 : Tính : $A=\frac{1}{6}-\frac{1}{35}-\frac{1}{63}-\frac{1}{99}-\frac{1}{143}-\frac{1}{195}$

Bài 25: Thực hiện phép tính: $A=\left(\frac{1}{4.9}+\frac{1}{9.14}+...+\frac{1}{44.49}\right).\frac{1-3-5-...-49}{89}$

HD:

Đặt : $B=\frac{1}{4.9}+\frac{1}{9.14}+...+\frac{1}{44.49}\Rightarrow 5B=\frac{5}{4.9}+\frac{5}{9.14}+...+\frac{5}{44.49}=\left(\frac{1}{4}-\frac{1}{49}\right)=\frac{45}{4.49}$

$\Rightarrow B=\frac{9}{4.49}$

và $C=\frac{1-3-5-...-49}{89}=\frac{1-(3+5+...+49)}{89}=\frac{1-612}{89}=\frac{-611}{89}$

Khi đó : $A=B.C=\frac{9}{4.49}.\frac{-611}{89}$

Bài 26: Thực hiện phép tính: $\frac{(1+2+3+...+100)(\frac{1}{2}-\frac{1}{3}-\frac{1}{7}-\frac{1}{9})(63.1,2-21.3,6)}{1-2+3-4+...+99-100}$

HD:

Ta có: $63.1,2-21.3,6=0\Rightarrow \frac{(1+2+3+...+100)(\frac{1}{2}-\frac{1}{3}-\frac{1}{7}-\frac{1}{9})(63.1,2-21.3,6)}{1-2+3-4+...+99-100}=0$

Bài 27: Tính tỉ số $\frac{A}{B}$ biết : $A=\frac{1}{1.300}+\frac{1}{2.301}+\frac{1}{3.302}+...+\frac{1}{101.400}$ và

$B=\frac{1}{1.102}+\frac{1}{2.103}+\frac{1}{3.104}+...+\frac{1}{299.400}$

HD:

$$299A=\frac{299}{1.300}+\frac{299}{2.301}+...+\frac{299}{101.400}=\left(\frac{1}{1}-\frac{1}{300}\right)+\left(\frac{1}{2}-\frac{1}{301}\right)+\left(\frac{1}{3}-\frac{1}{302}\right)+...+\left(\frac{1}{101}-\frac{1}{400}\right)$$

$$\Rightarrow 299A=\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{101}\right)-\left(\frac{1}{300}+\frac{1}{301}+...+\frac{1}{400}\right)$$

$$101B=\frac{101}{1.102}+\frac{101}{2.103}+\frac{101}{3.104}+...+\frac{101}{299.400}$$

$$=\left(1-\frac{1}{102}\right)+\left(\frac{1}{2}-\frac{1}{103}\right)+\left(\frac{1}{3}-\frac{1}{104}\right)+...+\left(\frac{1}{299}-\frac{1}{400}\right)$$

$$=\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{299}\right)-\left(\frac{1}{102}+\frac{1}{103}+...+\frac{1}{400}\right)=\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{101}\right)-\left(\frac{1}{300}+\frac{1}{301}+...+\frac{1}{400}\right)$$

Khi đó : $299A=101B\Rightarrow \frac{A}{B}=\frac{101}{299}$

DẠNG 5: TÍNH TỔNG TỰ NHIÊN DẠNG TÍCH

Bài 1: Tính nhanh các tổng sau

a, A= 1.2+2.3+3.4+…+98.99 b, B= $1.2+3.4+5.6+...+99.100$

HD:

a, Ta có: $3A=1.2(3-0)+2.3(4-1)+3.4(5-2)+...+98.99(100-97)$

$3A=(1.2.3-0.1.2)+(2.3.4-1.2.3)+(3.4.5-2.3.4)+...+(98.99.100-97.98.99)$

$3A=98.99.100 \Rightarrow A=\dfrac{98.99.100}{3}$

b, Ta có: $B=2+(2+1).4+(4+1).6+...+(98+1).100$

$B=2+(2.4+4)+(4.6+6)+...+(98.100+100)$

$B=(2.4+4.6+6.8+...+98.100)+(2+4+6+...+100)$

Đặt $M=2.4+4.6+6.8+...+98.100$

$6M=2.4(6-0)+4.6(8-2)+6.8(10-4)+...+98.100(102-96)$

$6M=(2.4.6-0.2.4)+(4.6.8-2.4.6)+(6.8.10-4.6.8)+...+(98.100.102-96.98.100)$

$6M=98.100.102 \Rightarrow M=\dfrac{98.100.102}{6}$

Tính $N=2+4+6+....+100$ rồi thay vào B

Bài 2: Tính nhanh các tổng sau

a, D= 1.4+2.5+3.6+…+100.103 b, E= $1.3+2.4+3.5+...+97.99+98.100$

HD:

a, Ta có:

$D=1.(1+3)+2.(2+3)+3(3+3)+...+100.(100+3)$

$D=(1.1+1.3)+(2.2+2.3)+(3.3+3.3)+...+(100.100+100.3)$

$D=(1.1+2.2+3.3+...+100.100)+3(1+2+3+...+100)$

Đặt, $A=1.1+2.2+3.3+...+100.100$ và $B=1+2+3+4+...+100$

b, Ta có:

$E=1(1+2)+2(2+2)+3(3+2)+...+97(97+2)+98(98+2)$

$E=(1.1+1.2)+(2.2+2.2)+(3.3+3.2)+...+(97.97+97.2)+(98.98+98.2)$

$E=(1.1+2.2+3.3+...+97.97+98.98)+2(1+2+3+4+...+97+98)$

Đặt $A=1.1+2.2+3.3+...+98.98$ và $B=1+2+3+4+...+97+98$

Tính rồi thay vào E

Bài 3: Tính nhanh các tổng sau

a, F= $1.3+5.7+9.11+...+97.101$ b, G= 1.2.3+2.3.4+3.4.5+…+98.99.100

HD:

a, $F=1.(1+2)+5(5+2)+9(9+2)+...+97(97+2)$

$F=(1.1+1.2)+(5.5+5.2)+(9.9+9.2)+...+(97.97+97.2)$

$F=(1.1+5.5+9.9+...+97.97)+2(1+5+9+...+97)$

Đặt $A=1.1+5.5+9.9+...+97.97, B=1+5+9+...+97$, Tính rồi thay vào F

b, $4G=1.2.3(4-0)+2.3.4(5-1)+3.4.5(6-2)+...+98.99.100(101-97)$

$4G=(1.2.3.4-0.1.2.3)+(2.3.4.5-1.2.3.4)+(3.4.5.6-2.3.4.5)+...+(98.99.100.101-97.98.99.100)$

$4G=98.99.100.101 \Rightarrow G=\dfrac{98.99.100.101}{4}$

Bài 4: Tính nhanh các tổng sau

a, H= $1.99+2.98+3.97+...+50.50$ b, K= $1.99+3.97+5.95+...+49.51$

HD:

a, $H=1.99+2.(99-1)+3.(99-2)+...+50(99-49)$

$H=1.99+(2.99-1.2)+(3.99-2.3)+...+(50.99-49.50)$

$H=(1.99+2.99+3.99+...+50.99)-(1.2+2.3+3.4+...+49.50)$

Đặt $A=99(1+2+3+...+50), B=1.2+2.3+3.4+...+49.50$

Tính A và B rồi thay vào H

b, $K=1.99+3(99-2)+5.(99-4)+...+49(99-48)$

$K=1.99+(3.99-2.3)+(5.99-4.5)+...+(49.99-48.49)$

$K=(1.99+3.99+5.99+...+49.99)-(2.3+4.5+...+48.49)$

Đặt $A=99(1+3+5+...+49), B=(2.3+4.5+6.7+...+48.49)$

Tính A và B rồi thay vào K

Bài 5: Tính nhanh các tổng sau : C= $1.3+3.5+5.7+...+97.99$

HD:

$C=1.(1+2)+3.(3+2)+5(5+2)+...+97.(97+2)$

$C=(1.1+1.2)+(3.3+3.2)+(5.5+5.2)+...+(97.97+97.2)$

$C=(1.1+3.3+...+97.97)+2(1+3+5+...+97)$

Đặt $A=1.1+3.3+5.5+...+97.97, B=1+3+5+7+...+97$

Tính A và B rồi thay vào C

Bài 6: Tính: $A=\dfrac{1.99+2.98+...+99.1}{1.2+2.3+3.4+...+99.100}$

DẠNG 6: TÍNH TỔNG CÔNG THỨC

Bài 1: Tính tổng: $D=1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+...+\frac{1}{20}(1+2+...+20)$

HD:

Ta có: $D=1+\frac{1}{2}.\frac{2.3}{2}+\frac{1}{3}.\frac{3.4}{2}+...+\frac{1}{20}.\frac{20.21}{2}=1+\frac{3}{2}+\frac{4}{2}+\frac{5}{2}+...+\frac{21}{2}$

$=\frac{1}{2}(2+3+4+...+20+21)=\frac{1}{2}.230=115$

Bài 2: Tính tổng: $F=1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+...+\frac{1}{2016}(1+2+...+2016)$

HD:

Ta có: $F=1+\frac{1}{2}.\frac{2.3}{2}+\frac{1}{3}.\frac{3.4}{2}+....+\frac{1}{2016}.\frac{2016.2017}{2}$

$F=1+\frac{2+1}{2}+\frac{3+1}{2}+\frac{4+1}{2}+...+\frac{2016+1}{2}=1+\frac{1}{2}+\frac{1}{2}+...+\frac{1}{2}+\frac{2+3+4+...+2016}{2}$

$F=1+\frac{1}{2}.2015+\frac{2018.2015}{2}=1+\frac{2015.2019}{2}$

Bài 3: Tính:$1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+...+\frac{1}{16}(1+2+...+16)$

HD:

Ta có: $F=1+\frac{1}{2}.\frac{2.3}{2}+\frac{1}{3}.\frac{3.4}{2}+....+\frac{1}{16}.\frac{16.17}{2}$

$F=1+\frac{2+1}{2}+\frac{3+1}{2}+\frac{4+1}{2}+...+\frac{16+1}{2}=1+\frac{1}{2}+\frac{1}{2}+...+\frac{1}{2}+\frac{2+3+4+...+16}{2}$

Bài 4: Tính tổng: $G=1+\frac{1}{2}(1+2)+\frac{1}{3}(1+2+3)+...+\frac{1}{100}(1+2+...+100)$

HD:

Ta có: $G=1+\frac{1}{2}.\frac{(1+2).2}{2}+\frac{1}{3}.\frac{(1+3).3}{2}+...+\frac{1}{100}.\frac{(1+100).100}{2}$

$G=1+\frac{2+1}{2}+\frac{3+1}{2}+\frac{4+1}{2}+...+\frac{100+1}{2}=1+\frac{1}{2}+\frac{1}{2}+...+\frac{1}{2}+\frac{2+3+4+...+100}{2}$

Bài 5: Tính tổng: $H=1+\frac{1}{2}.\frac{3.2}{2}+\frac{1}{3}.\frac{4.3}{2}+...+\frac{1}{500}.\frac{501.500}{2}$

HD:

Ta có: $H=1+\frac{3}{2}+\frac{4}{2}+\frac{5}{2}+...+\frac{501}{2}=1+\frac{3+4+5+...+501}{2}$

Bài 6: Tính: $\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+...+\frac{1}{1+2+...+59}$

HD:

Ta có: $=\frac{1}{\frac{(1+3).3}{2}}+\frac{1}{\frac{(1+4).4}{2}}+\frac{1}{\frac{(1+5).5}{2}}+...+\frac{1}{\frac{(1+59).59}{2}}$

$=\frac{2}{3.4}+\frac{2}{4.5}+\frac{2}{5.6}+...+\frac{2}{59.60}=2\left(\frac{1}{3.4}+\frac{1}{4.5}+\frac{1}{5.6}+...+\frac{1}{59.60}\right)=2\left(\frac{1}{3}-\frac{1}{60}\right)=2\left(\frac{19}{60}\right)=\frac{19}{30}$

Bài 7: Tính: $50+\frac{50}{3}+\frac{25}{3}+\frac{20}{4}+\frac{10}{3}+\frac{100}{6.7}+...+\frac{100}{98.99}+\frac{1}{99}$

HD:

Ta có: $A=\left(50+\frac{50}{3}+\frac{25}{3}+\frac{20}{4}+\frac{10}{3}\right)+\left(\frac{100}{6.7}+\frac{100}{7.8}+...+\frac{100}{98.99}+\frac{100}{99.100}\right)$

$A=100\left(\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\frac{1}{5.6}\right)+100\left(\frac{1}{6.7}+\frac{1}{7.8}+...+\frac{1}{99.100}\right)$

$A=100\left(\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{99.100}\right)=100.\left(1-\frac{1}{100}\right)=99$

DẠNG 7: TÍNH TÍCH

Bài 1: Tính tích

a, A= $\frac{2^2}{1.3}.\frac{3^2}{2.4}.\frac{4^2}{3.5}...\frac{20^2}{19.21}$ b, B= $\frac{1^2}{1.2}.\frac{2^2}{2.3}.\frac{3^2}{3.4}...\frac{10^2}{10.11}$

HD:

a, Ta có: $A=\frac{2.2}{1.3}.\frac{3.3}{2.4}.\frac{4.4}{3.5}....\frac{20.20}{19.21}=\frac{(2.3.4...20)(2.3.4...20)}{(1.2.3....19)(3.4.5...21)}=\frac{20.2}{21}=\frac{40}{21}$

b, Ta có: $B=\frac{1.1}{1.2}.\frac{2.2}{2.3}.\frac{3.3}{3.4}....\frac{10.10}{10.11}=\frac{(1.2.3....10)(1.2.3...10)}{(1.2.3...10)(2.3.4...11)}=\frac{1}{11}$

Bài 2: Tính tổng C = $\left(1-\frac{1}{1+2}\right)\left(1-\frac{1}{1+2+3}\right)\left(1-\frac{1}{1+2+3+4}\right)...\left(1-\frac{1}{1+2+3+...+2016}\right)$

HD:

Ta có: $C=\left(1-\frac{1}{\frac{(1+2).2}{2}}\right).\left(1-\frac{1}{\frac{(1+3).3}{2}}\right).\left(1-\frac{1}{\frac{(1+4).4}{2}}\right)....\left(1-\frac{1}{\frac{(1+2016).2016}{2}}\right)$

$=\frac{2}{3}.\frac{5}{6}.\frac{9}{10}.....\frac{2017.2016-2}{2016.2017}=\frac{4}{6}.\frac{10}{12}.\frac{18}{20}....\frac{2016.2017-2}{2016.2017}$

$C=\frac{1.4}{2.3}.\frac{2.5}{3.4}.\frac{3.6}{4.5}....\frac{2015.2018}{2016.2017}=\frac{1004}{3009}$

Bài 3: Tính: A = $\left(\frac{1}{2}-\frac{1}{3}\right)\left(\frac{1}{2}-\frac{1}{5}\right)\left(\frac{1}{2}-\frac{1}{7}\right)...\left(\frac{1}{2}-\frac{1}{99}\right)$

HD:

Ta có: $A=\frac{1}{2.3}.\frac{3}{2.5}.\frac{5}{2.7}....\frac{97}{2.99}=\frac{(1.3.5....97)}{2^{49}.(3.5.7...99)}=\frac{1}{2^{49}.99}$

Bài 4: Tính: $\frac{\left(1+\frac{1999}{1}\right)\left(1+\frac{1999}{2}\right)...\left(1+\frac{1999}{1000}\right)}{\left(1+\frac{1000}{1}\right)\left(1+\frac{1000}{2}\right)...\left(1+\frac{1000}{1999}\right)}$

HD:

Ta có: $A=\left(\frac{2000}{1}.\frac{2001}{2}.\frac{2002}{3}...\frac{2999}{1000}\right):\left(\frac{1001}{1}.\frac{1002}{2}.\frac{1003}{3}....\frac{2999}{1999}\right)$

$A=\left(\frac{2000.2001.2002...2999}{1.2.3.4...1000}\right).\left(\frac{1.2.3...1999}{1001.1002....2999}\right)=\frac{1001.1002....1999}{1001.1002...1999}=1$

Bài 5: Tính: $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)...\left(1-\frac{1}{400}\right)$

HD:

Ta có: $=\frac{3}{4}.\frac{8}{9}.\frac{15}{16}....\frac{399}{400}=\frac{1.3}{2.2}.\frac{2.4}{3.3}.\frac{3.5}{4.4}...\frac{19.21}{20.20}=\frac{(1.2.3...19)(3.4.5...21)}{(2.3.4...20)(2.3.4.5...20)}=\frac{21}{20.2}=\frac{21}{40}$

Bài 6: Tính: $\left(1-\frac{1}{1+2}\right)\left(1-\frac{1}{1+2+3}\right)...\left(1-\frac{1}{1+2+3+...+n}\right)$

HD:

Ta có: $A=\left(1-\frac{1}{\frac{(1+2).2}{2}}\right)\left(1-\frac{1}{\frac{(1+3).3}{2}}\right)...\left(1-\frac{1}{\frac{(1+n).n}{2}}\right)$

$=\left(1-\frac{2}{2.3}\right)\left(1-\frac{2}{3.4}\right)\left(1-\frac{2}{4.5}\right)....\left(1-\frac{2}{n(n+1)}\right)=\frac{4}{2.3}.\frac{10}{3.4}.\frac{18}{4.5}....\frac{n(n+1)-2}{n(n+1)}$

$=\frac{1.4}{2.3}.\frac{2.5}{3.4}.\frac{3.6}{4.5}...\frac{(n-1)(n+2)}{n(n+1)}=\frac{(1.2.3...(n-1))(4.5....(n+2))}{(2.3...n)(3.4.5...(n+1))}=\frac{n+2}{n.3}=\frac{n+2}{3n}$

Bài 7: Tính:

a/ $\left(1+\frac{1}{1.3}\right)\left(1+\frac{1}{2.4}\right)\left(1+\frac{1}{3.5}\right)...\left(1+\frac{1}{17.19}\right)$ b/ $\left(1-\frac{1}{21}\right)\left(1-\frac{1}{28}\right)\left(1-\frac{1}{36}\right)...\left(1-\frac{1}{1326}\right)$

HD:

a, Ta có:

$A=\frac{4}{1.3}.\frac{9}{2.4}.\frac{16}{3.5}....\frac{17.19+1}{17.19}=\frac{2.2}{1.3}.\frac{3.3}{2.4}.\frac{4.4}{3.5}...\frac{18.18}{17.19}=\frac{(2.3.4...18)(2.3.4...18)}{(1.2.3...17)(3.4.5...19)}=\frac{18.2}{19}=\frac{36}{19}$

b, Ta có:

$B=\frac{20}{21}.\frac{27}{28}.\frac{35}{36}....\frac{1325}{1326}=\frac{40}{42}.\frac{54}{56}.\frac{70}{72}....\frac{2650}{2652}=\frac{5.8}{6.7}.\frac{6.9}{7.8}.\frac{7.10}{8.9}...\frac{50.53}{51.52}$

$B=\frac{(5.6.7...50)(8.9.10...53)}{(6.7.8...51)(7.8.9...52)}=\frac{5.53}{51.7}$

Bài 8: Tính tích

a, D= $\frac{2^2}{3}.\frac{3^2}{8}.\frac{4^2}{15}.\frac{5^2}{24}.\frac{6^2}{35}.\frac{7^2}{48}.\frac{8^2}{63}.\frac{9^2}{80}$ b, E= $\frac{8}{9}.\frac{15}{16}.\frac{24}{25}...\frac{2499}{2500}$

HD:

a, $D=\frac{2.2}{1.3}.\frac{3.3}{2.4}.\frac{4.4}{3.5}....\frac{8.8}{7.9}.\frac{9.9}{8.10}=\frac{(2.3.4...8.9)(2.3.4...8.9)}{(1.2.3...7.8)(3.4.5...9.10)}=\frac{9.2}{10}=\frac{9}{5}$

b, $E=\frac{2.4}{3.3}.\frac{3.5}{4.4}.\frac{4.6}{5.5}....\frac{49.51}{50.50}=\frac{(2.3.4...49)(4.5.6...51)}{(3.4.5...50)(3.4.5...50)}=\frac{2.51}{50.3}=\frac{17}{25}$

Bài 9: Tính tích

a, G= $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)...\left(1-\frac{1}{100}\right)$ b, H= $\left(1-\frac{1}{7}\right)\left(1-\frac{2}{7}\right)\left(1-\frac{3}{7}\right)...\left(1-\frac{10}{7}\right)$

HD:

a, Ta có: $G=\frac{1}{2}.\frac{2}{3}.\frac{3}{4}....\frac{99}{100}=\frac{1}{100}$

b, Ta có: $H=\frac{6}{7}.\frac{5}{7}.\frac{4}{7}.\frac{3}{7}.\frac{2}{7}.\frac{1}{7}.\frac{0}{7}.\frac{-1}{7}.\frac{-2}{7}.\frac{-3}{7}=0$

Bài 10: Tính tích

a, I= $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{16}\right)...\left(1-\frac{1}{10000}\right)$ b, J= $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{1}{10}\right)...\left(1-\frac{1}{780}\right)$

HD:

a, Ta có: $I=\frac{3}{4}.\frac{8}{9}.\frac{15}{16}...\frac{9999}{10000}=\frac{1.3}{2.2}.\frac{2.4}{3.3}.\frac{3.5}{4.4}....\frac{99.101}{100.100}=\frac{(1.2.3....99)(3.4.5...101)}{(2.3.4...100)(2.3.4...100)}=\frac{101}{100.2}$

b, Ta có: $J=\frac{2}{3}.\frac{5}{6}.\frac{9}{10}...\frac{779}{780}$ $J=\frac{4}{6}.\frac{10}{12}.\frac{18}{20}....\frac{1558}{1560}=\frac{1.4}{2.3}.\frac{2.5}{3.4}.\frac{3.8}{4.5}....\frac{38.41}{39.40}$

$=\frac{(1.2.3...38)(4.5.6...40.41)}{(2.3.4...39)(3.4.5....40)}=\frac{41}{39.3}$

Bài 11: Tính tích

a, K= $\left(1-\frac{1}{21}\right)\left(1-\frac{1}{28}\right)\left(1-\frac{1}{36}\right)...\left(1-\frac{1}{1326}\right)$ b, M= $\left(\frac{1}{2}+1\right)\left(\frac{1}{3}+1\right)\left(\frac{1}{4}+1\right)...\left(\frac{1}{999}+1\right)$

HD:

a, Ta có: $K=\frac{20}{21}.\frac{27}{28}.\frac{35}{36}....\frac{1325}{1326}=\frac{40}{42}.\frac{54}{56}.\frac{70}{72}....\frac{2650}{2652}=\frac{5.8}{6.7}.\frac{6.9}{7.8}.\frac{7.10}{8.9}....\frac{50.53}{51.52}$

$\frac{(5.6.7...50)(8.9.10....53)}{(6.7.8...51)(7.8.9....52)}=\frac{5}{51}.\frac{53}{7}$

b, Ta có: $M=\frac{3}{2}.\frac{4}{3}.\frac{5}{4}....\frac{1000}{999}=\frac{1000}{2}=500$

Bài 12: Tính tích

a, F= $\frac{3}{2^2}.\frac{8}{3^2}.\frac{15}{4^2}...\frac{99}{10^2}$ b, N= $\left(\frac{1}{2}-1\right)\left(\frac{1}{3}-1\right)\left(\frac{1}{4}-1\right)...\left(\frac{1}{1000}-1\right)$

HD:

a, $F=\frac{1.3}{2.2}.\frac{2.4}{3.3}.\frac{3.5}{4.4}....\frac{9.11}{10.10}=\frac{(1.2.3...9)(3.4.5...11)}{(2.3.4...10)(2.3.4...10)}=\frac{1.11}{10.2}$

b, $N=\frac{-1}{2}.\frac{-2}{3}.\frac{-3}{4}....\frac{-999}{1000}=-\frac{1}{1000}$

Bài 13: Tính tích

a, C= $\frac{3}{4}.\frac{8}{9}.\frac{15}{16}...\frac{9999}{10000}$ b, $A=\left(\frac{1-2^2}{2^2}\right)\left(\frac{1-3^2}{3^2}\right)\left(\frac{1-4^2}{4^2}\right)...\left(\frac{1-2012^2}{2012^2}\right)$

HD:

a, Ta có: $C=\frac{1.3}{2.2}.\frac{2.4}{3.3}.\frac{3.5}{4.4}....\frac{99.101}{100.100}=\frac{(1.2.3...99)(3.4.5...101)}{(2.3.4...100)(2.3.4...100)}=\frac{1.101}{100.2}$

b, Ta có: $A=\frac{-3}{2.2}.\frac{-8}{3.3}.\frac{-15}{4.4}....\frac{1-2012^2}{2012.2012}=\frac{-1.3}{2.2}.\frac{-2.4}{3.3}.\frac{-3.5}{4.4}....\frac{-2011.2013}{2012.2012}$

$=-\frac{(1.2.3...2011)(3.4.5...2013)}{(2.3.4....2012)(2.3.4...2012)}=-\frac{2013}{2012.2}$

Bài 14: Tính giá trị của biểu thức: $C=\left(1+\frac{8}{10}\right)\left(1+\frac{8}{22}\right)\left(1+\frac{8}{36}\right)...\left(1+\frac{8}{8352}\right)$

HD:

Ta có : $C=\left(1+\frac{8}{10}\right)\left(1+\frac{8}{22}\right)\left(1+\frac{8}{36}\right)...\left(1+\frac{8}{8352}\right)=\frac{18}{10}.\frac{30}{22}.\frac{44}{36}....\frac{8360}{8352}=\frac{3.6}{2.5}.-$

Bài 15: Cho $E=\left(1-\frac{1}{1+2}\right)\left(1-\frac{1}{1+2+3}\right)...\left(1-\frac{1}{1+2+3+...+n}\right)$ và $F=\frac{n+2}{n}$, Tính $\frac{E}{F}$

HD:

Ta có: $E=\left(1-\frac{1}{\frac{(1+2).2}{2}}\right)\left(1-\frac{1}{\frac{(1+3).3}{2}}\right)...\left(1-\frac{1}{\frac{(1+n).n}{2}}\right)$

$$=\left(1-\frac{2}{2.3}\right)\left(1-\frac{2}{3.4}\right)\left(1-\frac{2}{4.5}\right)....\left(1-\frac{2}{n(n+1)}\right)=\frac{4}{2.3}.\frac{10}{3.4}.\frac{18}{4.5}....\frac{n(n+1)-2}{n(n+1)}$$

$$=\frac{1.4}{2.3}.\frac{2.5}{3.4}.\frac{3.6}{4.5}...\frac{(n-1)(n+2)}{n(n+1)}=\frac{(1.2.3...(n-1))(4.5....(n+2))}{(2.3...n)(3.4.5...(n+1))}=\frac{n+2}{n.3}=\frac{n+2}{3n}$$

Mà $\frac{n+2}{n}>\frac{n+2}{3n}=>F>E$

Bài 16: Tính: $K=\left(1-\frac{4}{1}\right)\left(1-\frac{4}{9}\right)\left(1-\frac{4}{25}\right)...\left(1-\frac{1}{(2n-1)^2}\right)$

Bài 17: Cho $G=\left(1+\frac{1}{2}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{16}\right)...\left(1+\frac{1}{2^{1024}}\right)$ và $H=\frac{1}{2^{2047}}$, Tính G + H

Bài 18: Tính: $A=\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right)\left(1-\frac{1}{9}\right)\left(1-\frac{1}{11}\right)\left(1-\frac{1}{2}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{1}{8}\right)\left(1-\frac{1}{10}\right)$

Bài 19: Tính: $A=\left(1+\frac{1}{100}\right)\left(1+\frac{1}{99}\right)\left(1+\frac{1}{98}\right)...\left(1+\frac{1}{2}\right)$

Bài 20: Tính nhanh: $B=\left(\frac{6}{8}+1\right)\left(\frac{6}{18}+1\right)\left(\frac{6}{30}+1\right)...\left(\frac{6}{10700}+1\right)$

Bài 21: Tính nhanh: E= $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2}+\frac{1}{3}\right)\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)...\left(1+\frac{1}{2}+...+\frac{1}{10}\right)$

Bài 22: So sánh : $U=\frac{1.3.5....39}{21.22.23...40}\&V=\frac{1}{2^{20}-1}$

HD:

$$U=\frac{1.3.5...37.39}{(21.23.25....39)(22.24.26....40)}=\frac{1.3.5.7...37.39}{(21.23.25....39)2^{10}(11.12.13....20)}$$

$$U=\frac{1.3.5...39}{2^{10}(21.23....39)(11.13...19)(12.14.16.18.20)}=\frac{1.3.5..39}{2^{10}.(11.13...39)2^5(6.7.8.9.10)}$$

$$U=\frac{1.3.5..39}{2^{15}(7.9.11....39).(6.8.10)}=\frac{1.3.5...39}{2^{15}.(7.9...39).2^5.3.5}=\frac{1.3.5..39}{2^{20}.3.5.7...39}=\frac{1}{2^{20}}$$

Mà $\frac{1}{2^{20}}<\frac{1}{2^{20}-1}=>U<V$

DẠNG 8 : TÍNH TỔNG CÙNG SỐ MŨ

Bài 1: Tổng cùng số mũ:

a, A= $1^2+2^2+3^2+...+98^2$ b, B= $-1^2+2^2-3^2+4^2-...-19^2+20^2$

HD:

a, Ta có : $A=1.1+2.2+3.3+...+98.98$

$\Rightarrow A=1(2-1)+2(3-1)+3(4-1)+...+98(99-1)$

$\Rightarrow A=(1.2+2.3+3.4+...+98.99)-(1+2+3+...+98)$

Đặt $B=1.2+2.3+3.4+...+98.99$, Tính tổng B ta được :

$3B=1.2(3-0)+2.3(4-1)+3.4(5-2)+...+98.99(100-97)$

$3B=(1.2.3-0.1.2)+(2.3.4-1.2.3)+(3.4.5-2.3.4)+...+(98.99.100-97.98.99)$

$3B=98.99.100-0.1.2=98.99.100\Rightarrow B=\dfrac{98.99.100}{3}$

Thay vào A ta được : $A=B+\dfrac{98.99}{2}=\dfrac{98.99.100}{3}+\dfrac{98.99}{2}$

b, Ta có : $B=-1^2+2^2-3^2+4^2-...-19^2+20^2 \Rightarrow B=-(1^2-2^2+3^2-4^2+...+19^2-20^2)$

$B=-\left[(1^2+2^2+3^2+...+19^2+20^2)-2(2^2+4^2+6^2+...+20^2)\right]$

$B=-\left[\left(\dfrac{20.21.22}{3}+\dfrac{20.21}{2}\right)-2.2^2(1^2+2^2+3^2+...+10^2)\right]$

$B=-20.22.7-20.7-8\left(\dfrac{10.11.12}{3}+\dfrac{10.11}{2}\right)=-20.7.23-8(10.11.4+5.11)$

Bài 2 : Tổng cùng số mũ :

a, D= $1^2+3^2+5^2+...+99^2$ b, E= $11^2+13^2+15^2+...+199^2$

HD:

a, Ta có : $D=(1^2+2^2+3^2+4^2+...+99^2+100^2)-(2^2+4^2+6^2+...+100^2)$

$\Rightarrow D=\left(\dfrac{100.101.102}{3}+\dfrac{100.101}{2}\right)-2^2(1^2+2^2+3^2+...+50^2)$

Đặt $A=1^2+2^2+3^2+...+50^2 \Rightarrow A=\dfrac{50.51.52}{3}+\dfrac{50.51}{2}$, Thay vào D ta được :

$D=100.101.34+50.101-4(50.52.17+25.51)$

b, Ta có : $E=11^2+12^2+13^2+14^2+15^2+...+199^2+200^2-(12^2+14^2+...+200^2)$

Đặt $A=11^2+12^2+13^2+...+200^2, B=12^2+14^2+...+200^2$

Tính ta được :

$A=11.11+12.12+13.13+...+200.200=11.(12-1)+12.(13-1)+...+200.(201-1)$

$\Rightarrow A=(11.12-11)+(12.13-12)+(13.14-13)+...+(200.201-200)$

$A=(11.12+12.13+13.14+...+200.201)-(11+12+13+...+200)$

$A=\left(\dfrac{200.201.202}{3}-\dfrac{10.11.12}{2}\right)-\left(\dfrac{211.190}{2}\right)$

Và $B=2^2(6^2+7^2+8^2+...+100^2)=4\left(\dfrac{100.101.102}{3}-\dfrac{5.6.7}{2}\right)-\left(\dfrac{106.95}{2}\right)$

Vậy $E=A-B$

Bài 3 : Tổng cùng số mũ :

a, C= $2^2+4^2+6^2+...+20^2$ b, F= $1^2+4^2+7^2+...+100^2$

HD:

a, Ta có : $C=2^2\left(1^2+2^2+3^2+...+10^2\right)$

Đặt $A=1^2+2^2+3^2+...+10^2=1.1+2.2+3.3+...+10.10$

$A=1.(2-1)+2.(3-1)+3.(4-1)+...+10.(11-1)$

$A=(1.2+2.3+3.4+...+10.11)-(1+2+3+...+10)=\frac{10.11.12}{3}-\frac{10.11}{2}$

b, Ta có : $F=1.1+4.4+7.7+10.10+...+100.100$

$F=1(4-3)+4(7-3)+7(10-3)+10(13-3)+...+100(103-3)$

$F=(1.4-1.3)+(4.7-3.4)+(7.10-3.7)+(10.13-10.3)+...+(100.103-100.3)$

$F=(1.4+4.7+7.10+10.13+...+100.103)-3(1+4+7+10+...+100)$

Đặt $A=1.4+4.7+7.10+...+100.103, B=1+4+7+10+...+100$

Tính $9A=1.4(9-0)+4.7(10-1)+7.10(13-4)+...+100.103(106-97)$

$9A=(1.4.9-0.1.4)+(4.7.10-1.4.7)+(7.10.13-4.7.10)+...+(100.103.106-97.100.103)$

$9A=1.4.9+(100.103.106-1.4.7)=>A=\frac{100.103.106+8}{9}$

Tính B rồi thay vào F ta được : $F=A-3B$

Bài 4 : Cho biết : $1^2+2^2+3^2+...+12^2=650$, Tính nhanh tổng sau : $2^2+4^2+6^2+...+24^2$

HD :

Ta có : $2^2+4^2+6^2+...+24^2=2^2\left(1^2+2^2+...+12^2\right)=4.650$

Bài 5 : Tổng cùng số mũ :

a, G= $1^2+3^2+5^2+...+99^2$ b, K= $1.2^2+2.3^2+3.4^2+...+99.100^2$

HD:

a, Ta có :

$G=1.1+3.3+5.5+7.7+...+99.99$

$G=1.(3-2)+3.(5-2)+5(7-2)+7(9-2)+...+99(101-2)$

$G=(1.3-1.2)+(3.5-2.3)+(5.7-2.5)+(7.9-2.7)+...+(99.101-2.99)$

$G=(1.3+3.5+5.7+7.9+...+99.101)-2(1+3+5+7+...+99)$

Đặt $A=1.3+3.5+5.7+...+99.101, B=1+3+5+7+...+99$

Tính $A=>6A=1.3(6-0)+3.5(7-1)+5.7(9-3)+...+99.101(103-97)$

$6A=(1.3.6-0.1.3)+(3.5.7-1.3.5)+(5.7.9-3.5.7)+...+(99.101.103-97.99.101)$

$6A=1.3.6+(99.101.103-1.3.5)=99.101.103+3=>A=\frac{99.101.103+3}{6}$

Tính tổng B rồi thay vào G

b, Ta có :

$K=1.2.2+2.3.3+3.4.4+...+99.100.100$

$K=1.2(3-1)+2.3(4-1)+3.4(5-1)+...+99.100(101-1)$

$K=(1.2.3-1.2)+(2.3.4-2.3)+(3.4.5-3.4)+...+(99.100.101-99.100)$

$K=(1.2.3+2.3.4+3.4.5+...+99.100.101)-(1.2+2.3+3.4+...+99.100)$

Đặt $A=1.2.3+2.3.4+3.4.5+...+99.100.101, B=1.2+2.3+3.4+...+99.100$

Tính $A=>4A=1.2.3(4-0)+2.3.4(5-1)+3.4.5(6-2)+...+99.100.101(102-98)$

$4A=(1.2.3.4-0.1.2.3)+(2.3.4.5-1.2.3.4)+(3.4.5.6-2.3.4.5)+...+(99.100.101.102-98.99.100.101)$

$4A=99.100.101.102 => A=\dfrac{99.100.101.102}{4}$

Tính B tương tự rồi thay vào K

Bài 6 : Tổng cùng số mũ :

a, H= $2^2+4^2+6^2+...+100^2$ b, I= $1.3^2+3.5^2+5.7^2+...+97.99^2$

HD:

a, Ta có :

$H=2^2(1^2+2^2+3^2+...+50^2)=4.A$

$A=1.1+2.2+3.3+4.4+...+50.50$

$A=1.(2-1)+2(3-1)+3(4-1)+4(5-1)+...+50(51-1)$

$A=(1.2-1)+(2.3-2)+(3.4-3)+...+(50.51-50)$

$A=(1.2+2.3+3.4+...+50.51)-(1+2+3+...+50)$

Tính tổng A ta được : $A=\dfrac{50.51.51}{3}-\dfrac{50.51}{2}$, Thay vào H ta được

b, Ta có :

I= $1.3^2+3.5^2+5.7^2+...+97.99^2$ => $I=1.3.3+3.5.5+5.7.7+...+97.99.99$

$I=1.3(5-2)+3.5.(7-2)+5.7(9-2)+...+97.99(101-2)$

$I=(1.3.5-1.3.2)+(3.5.7-3.5.2)+(5.7.9-5.7.2)+...+(97.99.101-97.99.2)$

$I=(1.3.5+3.5.7+5.7.9+...+97.99.101)-2(1.3+3.5+5.7+...+97.99)$

Đặt $A=1.3.5+3.5.7+5.7.9+...+97.99.101, B=1.3+3.5+5.7+...+97.99$

Ta có :

$8A=1.3.5.8+3.5.7(9-1)+5.7.9(11-3)+...+97.99.101(103-95)$

$8A=1.3.5.8+(3.5.7.9-1.3.5.7)+(5.7.9.11-3.5.7.9)+...+(97.99.101.103-95.97.99.101)$

$8A=1.3.5.8+97.99.101.103-1.3.5.7=97.99.101.103-15 => A=\dfrac{97.99.101.103-15}{8}$

Tương tự tính B rồi thay vào I

Bài 7: Tính: $A=1.3^3+3.5^3+5.7^3+...+49.51^3$

Bài 8: Tính: $B=1.99^2+2.98^2+3.97^2+...+49.51^2$

Bài 9: Biết : $1^3+2^3+...+10^3=3025$, Tính $A=2^3+4^3+...+20^3$

HD:

$A=2^3(1^3+2^3+...+10^3)$

Bài 10: Cho biết: $1^2+2^2+3^2+...+12^2=650$, Tính nhanh tổng sau: $2^2+4^2+6^2+...+24^2$

HD:

Ta có: $2^2+4^2+6^2+...+24^2=2^2(1^2+2^2+...+12^2)=4.650$

DẠNG 9: TỔNG CÙNG CƠ SỐ

Bài 1: Tổng cùng cơ số:

a, A= $1+3+3^2+3^3+...+3^{2000}$ b, B= $2+2^3+2^5+2^7+...+2^{2009}$

HD:

a, Ta có : $3A=3+3^2+3^3+3^4+...+3^{2000}+3^{2001}$

$\Rightarrow 3A-A=2A=(3-3)+(3^2-3^2)+...+(3^{2000}-3^{2000})+(3^{2001}-1)$

$\Rightarrow 2A=3^{2001}-1 \Rightarrow A=\dfrac{3^{2001}-1}{2}$

b, Ta có : $2^2B=2^3+2^5+2^7+...+2^{2009}+2^{2011}$

$\Rightarrow 4B-B=3B=(2^3-2^3)+(2^5-2^5)+...+(2^{2009}-2^{2009})+(2^{2011}-2)$

$\Rightarrow 3B=2^{2011}-2 \Rightarrow B=\dfrac{2^{2011}-2}{3}$

Bài 2: Tổng cùng cơ số:

a, C= $5+5^3+5^5+5^7+...+5^{101}$ b, D= $1+3^2+3^4+3^6+...+3^{100}$

HD:

a, Ta có : $5^2C=5^3+5^5+5^7+...+5^{101}+5^{103}$

$\Rightarrow 25C-C=24C=(5^3-5^3)+(5^5-5^5)+...+(5^{101}-5^{101})+(5^{103}-5)$

$\Rightarrow 24C=5^{103}-5 \Rightarrow C=\dfrac{5^{103}-5}{24}$

b, Ta có : $3^2D=3^2+3^4+3^6+...+3^{100}+3^{102}$

$\Rightarrow 9D-D=8D=(3^2-3^2)+(3^4-3^4)+...+(3^{100}-3^{100})+(3^{102}-1)$

$\Rightarrow 8D=3^{102}-1 \Rightarrow D=\dfrac{3^{102}-1}{8}$

Bài 3: Tổng cùng cơ số:

a, E= $7+7^3+7^5+...+7^{99}$ b, F= $1+5^2+5^4+5^6+...+5^{2016}$

HD:

a, Ta có : $7^2E=7^3+7^5+7^7+...+7^{99}+7^{101}$

$\Rightarrow 49E-E=48E=(7^3-7^3)+(7^5-7^5)+...+(7^{99}-7^{99})+(7^{101}-7)$

$\Rightarrow 48E=7^{101}-7 \Rightarrow E=\dfrac{7^{101}-7}{48}$

b, Ta có : $5^2F=5^2+5^4+5^6+...+5^{2016}+6^{2018}$

$25F-F=24F=(5^2-5^2)+(5^4-5^4)+...+(5^{2016}-5^{2016})+(5^{2018}-1)$

$24F=5^{2018}-1 \Rightarrow F=\dfrac{5^{2018}-1}{24}$

Bài 4: Tổng cùng cơ số: G= $1+2^2+2^4+2^6+...+2^{2016}$

HD:

Ta có :

$2^2G=2^2+2^4+2^6+...+2^{2016}+2^{2018}$

$4G-G=3G=(2^2-2^2)+(2^4-2^4)+...+(2^{2016}-2^{2016})+(2^{2018}-1) \Rightarrow 3G=2^{2018}-1 \Rightarrow G=\dfrac{2^{2018}-1}{3}$

Bài 5: Tổng cùng cơ số:

a, $M=2^{50}-2^{49}-2^{48}-...-2^2-2$ b, $N=3^{100}-3^{99}+3^{98}-3^{97}+...+3^2-3^1+1$

HD:

a, Ta có :

$M=2^{50}-(2+2^2+2^3+...+2^{48}+2^{49})$

Đặt $A=2+2^2+2^3+2^4+...+2^{48}+2^{49}$, Tính A ta được :

$A=2^{50}-2$, Thay vào M ta được :

$M=2^{50}-A=2^{50}-\left(2^{50}-2\right)=2$

b, Ta có :

$N=1-3+3^2-3^3+...+9^{98}-9^{99}+3^{100}$

$=>3N=3-3^2+3^3-3^4+...+3^{99}-3^{100}+3^{101}$

$=>N+3N=\left(3-3\right)+\left(3^2-3^2\right)+\left(3^3-3^3\right)+...+\left(3^{100}-3^{100}\right)+3^{101}+1$

$4N=3^{101}+1=>N=\dfrac{3^{101}+1}{4}$

Bài 6: Tổng cùng cơ số : I= $1+2^2+2^3+...+2^{63}$

HD:

Ta có : $2I=2+2^3+2^4+...+2^{64}$

$=>2I-I=\left(2^3-2^3\right)+\left(2^4-2^4\right)+...+\left(2^{63}-2^{63}\right)+\left(2^{64}+2-1\right)=>I=2^{64}+1$

Bài 7: Tính giá trị của biểu thức: $B=1-2+2^2-2^3+...+2^{2008}$

HD:

Ta có : $B=1-2+2^2-2^3+...+2^{2008}=>2B=2-2^2+2^3-2^4+....+2^{2009}$

$=>2B+B=3B=1+2^{2009}=>B=\dfrac{2^{2009}+1}{3}$

Bài 8: Tính $A=2000\left(2001^9+2001^8+...+2001^2+2001\right)+1$

HD:

Đặt : $B=2001+2001^2+2001^3+...+2001^9=>2001B=2001^2+2001^3+...+2001^{10}$

$=>2001B-B=2000B=2001^{10}-2001$, Khi đó :

$A=2000B+1=2001^{10}-2001+1=2001^{10}-2000$

Bài 9: Cho $H=2^{2010}-2^{2009}-2^{2008}-...-2-1$, Tính 2010^H

HD:

Ta có : $H=2^{2010}-\left(1+2+2^2+2^3+...+2^{2008}+2^{2009}\right)$. Đặt : $A=1+2+2^2+2^3+...+2^{2009}$

Tính tổng A ta được : $A=2^{2010}-1$, Thay vào H ta được :

$H=2^{2010}-\left(2^{2010}-1\right)=1=>2010^H=2010$

Bài 10: Tính tổng : $S=\left(-3\right)^0+\left(-3\right)^1+\left(-3\right)^2+...+\left(-3\right)^{2015}$

Bài 11: Tính: $A=1+7+7^2+7^3+7^4+...+7^{2007}$

Bài 12: Tính $B=1+4+4^2+4^3+...+4^{100}$

Bài 13: Tổng cùng cơ số : H= $1+2.6+3.6^2+4.6^3+...+100.6^{99}$

HD :

Ta có : $6H=6+2.6^2+3.6^3+4.6^4+...+100.6^{100}$

$H-6H=-5H=\left(2.6-6\right)+\left(3.6^2-2.6^2\right)+\left(4.6^3-3.6^3\right)+...+\left(100.6^{99}-99.6^{99}\right)+\left(1-100.6^{100}\right)$

$-5H=6+6^2+6^3+...+6^{99}+\left(1-100.6^{100}\right)$

Đặt $A=6+6^2+6^3+...+6^{99}$, Tính A ta được :

$A=\dfrac{6^{100}-6}{5}$, Thay vào H ta được :

$-5H=A+\left(1-100.6^{100}\right)=\dfrac{6^{100}-6}{5}+1-100.6^{100}=\dfrac{6^{100}-6+5-500.6^{100}}{5}=-\dfrac{499.6^{100}+1}{5}$

$\Rightarrow H=\dfrac{499.6^{100}+1}{25}$

Bài 14: Tính tổng cơ số: A= $\dfrac{1}{7}+\dfrac{1}{7^2}+\dfrac{1}{7^3}+...+\dfrac{1}{7^{100}}$

HD:

a, Ta có: $\dfrac{1}{7}A=\dfrac{1}{7^2}+\dfrac{1}{7^3}+\dfrac{1}{7^4}+...+\dfrac{1}{7^{100}}+\dfrac{1}{7^{101}}$

$A-\dfrac{1}{7}A=\left(\dfrac{1}{7^2}-\dfrac{1}{7^2}\right)+\left(\dfrac{1}{7^3}-\dfrac{1}{7^3}\right)+...+\left(\dfrac{1}{7^{100}}-\dfrac{1}{7^{100}}\right)+\left(\dfrac{1}{7}-\dfrac{1}{7^{101}}\right)\Rightarrow \dfrac{6}{7}.A=\dfrac{7^{100}-1}{7^{101}}\Rightarrow A=\dfrac{7^{100}-1}{6.7^{100}}$

Bài 15: Tính tổng cơ số: B= $\dfrac{1}{3}+\dfrac{1}{3^2}+\dfrac{1}{3^3}+...+\dfrac{1}{3^{20}}$

HD :

Ta có: $\dfrac{1}{3}B=\dfrac{1}{3^2}+\dfrac{1}{3^3}+\dfrac{1}{3^4}+...+\dfrac{1}{3^{20}}+\dfrac{1}{3^{21}}$

$B-\dfrac{1}{3}B=\left(\dfrac{1}{3^2}-\dfrac{1}{3^2}\right)+\left(\dfrac{1}{3^3}-\dfrac{1}{3^3}\right)+...+\left(\dfrac{1}{3^{20}}-\dfrac{1}{3^{20}}\right)+\left(\dfrac{1}{3}-\dfrac{1}{3^{21}}\right)\Rightarrow \dfrac{2}{3}.B=\dfrac{3^{20}-1}{3^{21}}\Rightarrow B=\dfrac{3^{20}-1}{2.3^{20}}$

Bài 16: Tính tổng cơ số

a, D= $\left(-\dfrac{1}{7}\right)^0+\left(-\dfrac{1}{7}\right)^1+\left(-\dfrac{1}{7}\right)^2+...+\left(-\dfrac{1}{7}\right)^{2017}$ b, E= $-\dfrac{1}{3}+\dfrac{1}{3^2}-\dfrac{1}{3^3}+\dfrac{1}{3^4}-...+\dfrac{1}{3^{50}}-\dfrac{1}{3^{51}}$

HD:

a, Ta có: $D=1-\dfrac{1}{7}+\dfrac{1}{7^2}-\dfrac{1}{7^3}+...+\dfrac{1}{7^{2016}}-\dfrac{1}{7^{2017}}$

$\dfrac{1}{7}D=\dfrac{1}{7}-\dfrac{1}{7^2}+\dfrac{1}{7^3}-\dfrac{1}{7^4}+...+\dfrac{1}{7^{2017}}-\dfrac{1}{7^{2018}}$

$D+\dfrac{1}{7}D=\left(\dfrac{-1}{7}+\dfrac{1}{7}\right)+\left(\dfrac{1}{7^2}+\dfrac{-1}{7^2}\right)+...+\left(\dfrac{-1}{7^{2017}}+\dfrac{1}{7^{2017}}\right)+\left(1-\dfrac{1}{7^{2018}}\right)$

$\dfrac{8}{7}D=\dfrac{7^{2018}-1}{7^{2018}}\Rightarrow D=\dfrac{7^{2018}-1}{8.7^{2018}}$

b, Ta có: $\dfrac{1}{3}E=\dfrac{-1}{3^2}+\dfrac{1}{3^3}-\dfrac{1}{3^4}+...+\dfrac{1}{3^{51}}-\dfrac{1}{3^{52}}$

$E+\dfrac{1}{3}E=\left(\dfrac{1}{3^2}+\dfrac{-1}{3^2}\right)+\left(\dfrac{-1}{3^3}+\dfrac{1}{3^3}\right)+...+\left(\dfrac{-1}{3^{51}}+\dfrac{1}{3^{51}}\right)+\left(\dfrac{-1}{3}+\dfrac{-1}{3^{52}}\right)\Rightarrow \dfrac{4}{3}E=-\dfrac{3^{51}+1}{3^{52}}\Rightarrow E=-\dfrac{3^{51}+1}{4.3^{51}}$

Bài 17: Tính tổng cơ số G= $\dfrac{3}{5}+\dfrac{3}{5^4}+\dfrac{3}{5^7}+...+\dfrac{3}{5^{100}}$

HD:

Ta có: $G=\dfrac{3}{5}+\dfrac{3}{5^4}+\dfrac{3}{5^7}+...+\dfrac{3}{5^{100}}$ $G=3\left(\dfrac{1}{5}+\dfrac{1}{5^4}+\dfrac{1}{5^7}+...+\dfrac{1}{5^{100}}\right)$

Đặt $A=\dfrac{1}{5}+\dfrac{1}{5^4}+\dfrac{1}{5^7}+...+\dfrac{1}{5^{100}}\Rightarrow \dfrac{1}{5^3}A=\dfrac{1}{5^4}+\dfrac{1}{5^7}+\dfrac{1}{5^{10}}+...+\dfrac{1}{5^{103}}$

$A-\dfrac{1}{125}A=\left(\dfrac{1}{5^4}-\dfrac{1}{5^4}\right)+\left(\dfrac{1}{5^7}-\dfrac{1}{5^7}\right)+...+\left(\dfrac{1}{5^{100}}-\dfrac{1}{5^{100}}\right)+\left(\dfrac{1}{5}-\dfrac{1}{5^{103}}\right)$

$\dfrac{124.A}{125}=\dfrac{1}{5}-\dfrac{1}{5^{103}}=\dfrac{5^{102}-1}{5^{103}}\Rightarrow A=\dfrac{5^{102}-1}{5^{100}.124}$

Bài 18: Tính tổng cơ số

a, $K=\dfrac{200-\left(3+\dfrac{2}{3}+\dfrac{2}{4}+...+\dfrac{2}{100}\right)}{\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{3}{4}+...+\dfrac{99}{100}}=2$

b, $I=1+\dfrac{1}{2}+\left(\dfrac{1}{2}\right)^2+\left(\dfrac{1}{2}\right)^3+...+\left(\dfrac{1}{2}\right)^{100}$

HD:

a, Ta có: $TS=\left(2-\dfrac{2}{3}\right)+\left(2-\dfrac{2}{4}\right)+\left(2-\dfrac{2}{5}\right)+...+\left(2-\dfrac{2}{100}\right)+1$

$TS=\dfrac{4}{3}+\dfrac{6}{4}+\dfrac{8}{5}+...+\dfrac{198}{100}+\dfrac{2}{2}=2\left(\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{3}{4}+...+\dfrac{99}{100}\right)=2.MS \Rightarrow K=\dfrac{TS}{MS}=\dfrac{2MS}{MS}=2$

b, Ta có : $I=1+\dfrac{1}{2}+\dfrac{1}{2^2}+\dfrac{1}{2^3}+\dfrac{1}{2^4}+...+\dfrac{1}{2^{100}} \Rightarrow \dfrac{1}{2}I=\dfrac{1}{2}+\dfrac{1}{2^2}+\dfrac{1}{2^3}+...+\dfrac{1}{2^{100}}+\dfrac{1}{2^{101}}$

$\Rightarrow I-\dfrac{1}{2}I=\left(\dfrac{1}{2}-\dfrac{1}{2}\right)+\left(\dfrac{1}{2^2}-\dfrac{1}{2^2}\right)+\left(\dfrac{1}{2^3}-\dfrac{1}{2^3}\right)+...+\left(\dfrac{1}{2^{100}}-\dfrac{1}{2^{100}}\right)+\left(1-\dfrac{1}{2^{101}}\right)$

$\dfrac{1}{2}I=\dfrac{2^{101}-1}{2^{101}} \Rightarrow I=\dfrac{2^{101}-1}{2^{100}}$

Bài 19: Tính tổng cơ số: C= $\dfrac{1}{2}+\dfrac{1}{2^3}+\dfrac{1}{2^5}+...+\dfrac{1}{2^{99}}$

HD:

a, Ta có : $\dfrac{1}{2^2}C=\dfrac{1}{2^3}+\dfrac{1}{2^5}+\dfrac{1}{2^7}+...+\dfrac{1}{2^{99}}+\dfrac{1}{2^{101}}$

$C-\dfrac{1}{4}C=\dfrac{3}{4}C=\left(\dfrac{1}{2^3}-\dfrac{1}{2^3}\right)+\left(\dfrac{1}{2^5}-\dfrac{1}{2^5}\right)+...+\left(\dfrac{1}{2^{98}}-\dfrac{1}{2^{98}}\right)+\left(\dfrac{1}{2}-\dfrac{1}{2^{101}}\right)$

$\Rightarrow \dfrac{3}{4}C=\dfrac{2^{100}-1}{2^{101}} \Rightarrow C-\dfrac{2^{100}-1}{3.2^{99}}$

Bài 20: Tính:

a, $4.5^{100}\left(\dfrac{1}{5}+\dfrac{1}{5^2}+...+\dfrac{1}{5^{100}}\right)+1$

b, $1+\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+...+\dfrac{1}{1024}$

Bài 21: Tính $A=-\dfrac{1}{3}+\dfrac{1}{3^2}-\dfrac{1}{3^3}+\dfrac{1}{3^4}-...+\dfrac{1}{3^{50}}-\dfrac{1}{3^{51}}$

Bài 22: Tính tổng cơ số: H= $\dfrac{1}{3}+\dfrac{2}{3^2}+\dfrac{3}{3^3}+\dfrac{4}{3^4}+...+\dfrac{2017}{3^{2017}}$

HD :

Ta có : $\dfrac{1}{3}H=\dfrac{1}{3^2}+\dfrac{2}{3^3}+\dfrac{3}{3^4}+\dfrac{4}{3^5}+...+\dfrac{2016}{3^{2017}}+\dfrac{2017}{3^{2018}}$

$H-\dfrac{1}{3}H=\left(\dfrac{2}{3^2}-\dfrac{1}{3^2}\right)+\left(\dfrac{3}{3^3}-\dfrac{2}{3^3}\right)+\left(\dfrac{4}{3^4}-\dfrac{3}{3^4}\right)+...+\left(\dfrac{2017}{3^{2017}}-\dfrac{2016}{3^{2017}}\right)+\left(\dfrac{1}{3}-\dfrac{2017}{3^{2018}}\right)$

$\dfrac{2}{3}H=\dfrac{1}{3^2}+\dfrac{1}{3^3}+\dfrac{1}{3^4}+...+\dfrac{1}{3^{2017}}+\dfrac{1}{3}-\dfrac{2017}{3^{2018}}$

Đặt $A=\dfrac{1}{3}+\dfrac{1}{3^2}+\dfrac{1}{3^3}+...+\dfrac{1}{3^{2017}}$, Tính A rồi thay vào H

Bài 23: Tính tổng cơ số: F= $1+\dfrac{3}{2^3}+\dfrac{4}{2^4}+\dfrac{5}{2^5}+...+\dfrac{100}{2^{100}}$

HD:

Ta có: $\dfrac{1}{2}F=\dfrac{1}{2}+\dfrac{3}{2^4}+\dfrac{4}{2^5}+\dfrac{5}{2^6}+...+\dfrac{99}{2^{100}}+\dfrac{100}{2^{101}}$

$F-\dfrac{1}{2}F=\left(\dfrac{4}{2^4}-\dfrac{3}{2^4}\right)+\left(\dfrac{5}{2^5}-\dfrac{4}{2^5}\right)+...+\left(\dfrac{100}{2^{100}}-\dfrac{99}{2^{100}}\right)+\left(1+\dfrac{3}{2^3}-\dfrac{1}{2}-\dfrac{100}{2^{101}}\right)$

$$\frac{1}{2}F=\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}+...+\frac{1}{2^{100}}+\left(\frac{1}{2}-\frac{3}{8}-\frac{100}{2^{101}}\right)$$

Đặt $A=\frac{1}{2^4}+\frac{1}{2^5}+\frac{1}{2^6}+...+\frac{1}{2^{100}}$. Tính A rồi thay vào F

Bài 24: Tính: $A=4.5^{100}\left(\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+...+\frac{1}{5^{100}}\right)+1$

Bài 25: Cho $A=3+3^2+3^3+...+3^{2015}+3^{2016}$

a, Tính A

b, Tìm chữ số tận cùng của A

c, A có là số chính phương không

HD:

a, $A=\frac{3^{2017}-3}{2}$

b,

$A=\left(3+3^2+3^3+3^4\right)+...+\left(3^{2013}+3^{2014}+3^{2015}+3^{2016}\right)=3\left(1+3+3^2+3^3\right)+...+3^{2013}\left(1+3+3^2+3^3\right)$

$=3.40+...+3^{2013}.40=40\left(3+3^5+...+3^{2013}\right)$ nên A có tận cùng là 0

c, Lập luận được A chia hết cho 3

Lập luận được A không chia hết cho 3^2

Mà 3 là số nguyên tố nên A không là số chính phương

Bài 26: Chứng tỏ rằng : $M=75\left(4^{2017}+4^{2016}+...+4^2+4+1\right)+25$ chia hết cho 100

HD:

Tính tổng $M=75.\frac{4^{2018}-1}{3}+25=25.4^{2018}\vdots 100$

DẠNG 10: TÍNH ĐƠN GIẢN

Bài 1: Thực hiện phép tính:

a, $\dfrac{1.2.3+2.4.6+4.8.12+7.14.21}{1.3.5+2.6.10+4.12.20+7.21.35}$ b, $\dfrac{1.7.9+3.21.27+5.35.45+7.49.63}{1.3.5+3.9.15+5.15.25+7.21.35}$

HD:

a, Ta có : $\dfrac{1.2.3+2.4.6+4.8.12+7.14.21}{1.3.5+2.6.10+4.12.20+7.21.35}=\dfrac{1.2.3\left(1+2.2.2+4.4.4+7.7.7\right)}{1.3.5\left(1+2.2.2+4.4.4+7.7.7\right)}=\dfrac{1.2.3}{1.3.5}=\dfrac{2}{5}$

b, Ta có : $\dfrac{1.7.9+3.21.27+5.35.45+7.49.63}{1.3.5+3.9.15+5.15.25+7.21.35}=\dfrac{1.7.9\left(1+3.3.3+5.5.5+7.7.7\right)}{1.3.5\left(1+3.3.3+5.5.5+7.7.7\right)}=\dfrac{1.7.9}{1.3.5}=\dfrac{21}{5}$

Bài 2: Thực hiện phép tính: $\dfrac{1.2+2.4+3.6+4.8+5.10}{3.4+6.8+9.12+12.16+15.20}$

HD:

Ta có : $\dfrac{1.2+2.4+3.6+4.8+5.10}{3.4+6.8+9.12+12.16+15.20}=\dfrac{1.2\left(1+2.2+3.3+4.4+5.5\right)}{3.4\left(1+2.2+3.3+4.4+5.5\right)}=\dfrac{1.2}{3.4}=\dfrac{1}{6}$

Bài 3: Tính:

a, $A=\dfrac{1.2+2.4+4.8+7.14}{1.3+2.6+4.12+7.21}$ b, $B=\dfrac{2.3+4.6+6.9+8.12}{3.4+6.8+9.12+12.16}$

Bài 4: Tính giá trị của biểu thức sau: $B=\dfrac{2a}{5b}+\dfrac{5b}{6c}+\dfrac{6c}{7d}+\dfrac{7d}{2a}$ biết $\dfrac{2a}{5b}=\dfrac{5b}{6c}=\dfrac{6c}{7d}=\dfrac{7d}{2a}$

và a, b, c, d # 0

HD:

Đặt $B=\dfrac{2a}{5b}=\dfrac{5b}{6c}=\dfrac{6c}{7d}=\dfrac{7d}{2a}=k\Rightarrow \dfrac{2a}{5b}.\dfrac{5b}{6c}.\dfrac{6c}{7d}.\dfrac{7d}{2a}=k^4=1\Rightarrow k=\pm 1\Rightarrow B=\pm 4$

Bài 5: Tính gá trị của biểu thức: $\dfrac{2a}{3b}+\dfrac{3b}{4c}+\dfrac{4c}{5d}+\dfrac{5d}{2a}$ biết $\dfrac{2a}{3b}=\dfrac{3b}{4c}=\dfrac{4c}{5d}=\dfrac{5d}{2a}$

HD:

Đặt : $\dfrac{2a}{3b}=\dfrac{3b}{4c}=\dfrac{4c}{5d}=\dfrac{5d}{2a}=k\Rightarrow k^4=\dfrac{2a}{3b}.\dfrac{3b}{4c}.\dfrac{4c}{5d}.\dfrac{5d}{2a}=1\Rightarrow k=\pm 1$

Khi đó : $\dfrac{2a}{3b}+\dfrac{3b}{4c}+\dfrac{4c}{5d}+\dfrac{5d}{2a}=1$ hoặc $\dfrac{2a}{3b}+\dfrac{3b}{4c}+\dfrac{4c}{5d}+\dfrac{5d}{2a}=-1$

Bài 6: Tính gá trị của biểu thức: B= $\dfrac{a^2m-a^2n-b^2n+b^2m}{a^2+b^2}$

HD :

Ta có : $B=\dfrac{a^2\left(m-n\right)+b^2\left(m-n\right)}{a^2+b^2}=\dfrac{\left(a^2+b^2\right)\left(m-n\right)}{\left(a^2+b^2\right)}=m-n$

Bài 7: Thực hiện phép tính: $\dfrac{\left(ab+bc+cd+da\right)abcd}{\left(c+d\right)\left(a+b\right)+\left(b-c\right)\left(a-d\right)}$

HD:

Ta có : $MS=ca+cb+da+bd+ab-bd-ca+cd=\left(ab+bc+cd+da\right)$

Khi đó : $\dfrac{TS}{MS}=\dfrac{(ab+bc+cd+da)abcd}{\left(ab+bc+cd+da\right)}=abcd$

Bài 8: Tính giá trị của biểu thức: $A=\dfrac{\left(a+b\right)\left(-x-y\right)-\left(a-y\right)\left(b-x\right)}{abxy\left(xy+ay+ab+bx\right)}$

HD:

Ta có: $TS=-ax-ay-bx-by-ab+ax+yb-xy=-\left(ay+ab+bx+xy\right)$

Khi đó: $A=\dfrac{-(ay+ab+bx+xy)}{abxy(ay+ab+bx+xy)}=\dfrac{-1}{abxy}$

Bài 9: Tính tổng

a, A= $\dfrac{2^0+2^1+2^2+...+2^{2004}}{1+2^5+2^{10}+...+2^{2000}}$ b, B= $\dfrac{1+5+5^2+5^3+...+5^{100}}{1+4+4^2+4^3+...+4^{100}}$

HD:

a, Ta có: $A=\dfrac{(1+2+2^2+2^3+2^4)+(2^5+2^6+2^7+2^8+2^9)+...+(2^{2000}+2^{2001}+2^{2002}+2^{2003}+2^{2004})}{1+2^5+2^{10}+2^{15}+...+2^{2000}}$

$A=\dfrac{(1+2+2^2+2^3+2^4)+2^5(1+2+2^2+2^3+2^4)+...+2^{2000}(1+2+2^2+2^3+2^4)}{1+2^5+2^{10}+2^{15}+...+2^{2000}}$

$A=\dfrac{(1+2+2^2+2^3+2^4)(1+2^5+2^{10}+...+2^{2000})}{(1+2^5+2^{10}+...+2^{2000})}=(1+2+2^2+2^3+2^4)$

b, Ta có: $M=1+5+5^2+5^3+...+5^{100}$

$\Rightarrow 5M=5+5^2+5^3+...+5^{100}+5^{101} \Rightarrow 5M-M=4M=5^{101}-1\Rightarrow M=\dfrac{5^{101}-1}{4}$

và $N=1+4+4^2+4^3+...+4^{100}$

$\Rightarrow 4N=4+4^2+4^3+4^4+...+4^{101}\Rightarrow 4N-N=3N=4^{101}-1\Rightarrow N=\dfrac{4^{101}-1}{3}$

Khi đó: $B=\dfrac{M}{N}$

Bài 10: Thu gọn biểu thức: $A=\dfrac{x^{95}+x^{94}+x^{93}+...+x+1}{x^{31}+x^{30}+x^{29}+...+x+1}$

Bài 11: Tính tổng: A= $\dfrac{101+100+99+...+2+1}{101-100+99-98+...-2+1}$

HD:

Ta có: $TS=\dfrac{(1+101).101}{2}=101.51=5151$

$MS=(101-100)+(99-98)+...+(3-2)+1=1+1+...+1=51$. Khi đó: $A=\dfrac{TS}{MS}=\dfrac{51.101}{51}=101$

Bài 12: Tính: $\dfrac{1.99+2.98+...+99.1}{1.2+2.3+...+99.100}$

HD:

Ta có: $TS=1.99+2.(99-1)+3.(99-2)+...+99(99-98)$

$=1.99+(2.99-1.2)+(3.99-2.3)+...+(99.99-98.99)$

$=99(1+2+3+...+99)-(1.2+2.3+3.4+...+98.99)$

Đặt $A=1+2+3+...+99, B=1.2+2.3+3.4+...+98.99$, Tính A và B rồi thay vào ta được:

Bài 13: Thực hiện phép tính:

a, A= $\dfrac{120-\frac{1}{2}.40.5.\frac{1}{5}.20.\frac{1}{4}-20}{1+5+9+...+33+37+41}$

HD:

a, Ta có: $TS=120-20.5-20=0$, Khi đó $A=0$

DẠNG 11: TÍNH TỈ SỐ CỦA HAI TỔNG

Bài 1: Thực hiện phép tính: $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2012}}{\frac{2011}{1}+\frac{2010}{2}+\frac{2009}{3}+...+\frac{1}{2011}}$

HD:

Mẫu số : $MS=\left(1+\frac{2010}{2}\right)+\left(1+\frac{2009}{3}\right)+...+\left(1+\frac{1}{2011}\right)+1=\frac{2012}{2}+\frac{2012}{3}+...+\frac{2012}{2011}+\frac{2012}{2012}$

$MS=2012\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2012}\right)=2012.TS$

Khi đó : $A=\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2012}}{\frac{2011}{1}+\frac{2010}{2}+\frac{2009}{3}+...+\frac{1}{2011}}=\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2012}}{2012\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2012}\right)}=\dfrac{1}{2012}$

Bài 2: Thực hiện phép tính: $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{99}+\frac{1}{100}}{\frac{99}{1}+\frac{98}{2}+\frac{97}{3}+...+\frac{1}{99}}$

HD:

$MS=\left(1+\frac{98}{2}\right)+\left(1+\frac{97}{3}\right)+...+\left(1+\frac{1}{99}\right)+1=\frac{100}{2}+\frac{100}{3}+...+\frac{100}{99}+\frac{100}{100}$

$MS=100\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{100}\right)=100.TS$

Khi đó : $A=\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{100}}{\frac{99}{1}+\frac{98}{2}+\frac{97}{3}+...+\frac{1}{99}}=\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{100}}{100\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{100}\right)}=\dfrac{1}{100}$

Bài 3: Tính tỉ số $\dfrac{A}{B}$ biết : $A=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2009}$ và $B=\frac{2008}{1}+\frac{2007}{2}+\frac{2006}{3}+...+\frac{1}{2008}$

HD:

Ta có : $B=\left(1+\frac{2007}{2}\right)+\left(1+\frac{2006}{3}\right)+...+\left(1+\frac{1}{2008}\right)+1=\frac{2009}{2}+\frac{2009}{3}+\frac{2009}{4}+...+\frac{2009}{2008}+\frac{2009}{2009}$

$=2009\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2008}+\frac{1}{2009}\right)=2009.A$

Khi đó : $\dfrac{A}{B}=\dfrac{A}{2009A}=\dfrac{1}{2009}$

Bài 4: Tính tỉ số $\dfrac{A}{B}$ biết: $A=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{200}$ và $B=\frac{1}{199}+\frac{2}{198}+\frac{3}{197}+...+\frac{198}{2}+\frac{199}{1}$

HD:

Ta có : $B=\left(1+\frac{1}{199}\right)+\left(1+\frac{2}{198}\right)+\left(1+\frac{3}{197}\right)+...+\left(1+\frac{198}{2}\right)+1=\frac{200}{199}+\frac{200}{198}+...+\frac{200}{2}+\frac{200}{200}$

$B=200\left(\frac{1}{199}+\frac{1}{198}+...+\frac{1}{2}+\frac{1}{200}\right)=200.A \Rightarrow \dfrac{A}{B}=\dfrac{1}{200}$

Bài 5: Tính tỉ số $\dfrac{A}{B}$ biết : $A=\frac{1}{2012}+\frac{2}{2011}+...+\frac{2011}{2}+\frac{2011}{1}$ và $B=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2013}$

HD:

Ta có : $A=\left(\frac{1}{2012}+1\right)+\left(\frac{2}{2011}+1\right)+...+\left(\frac{2011}{2}+1\right)+1=\frac{2013}{2012}+\frac{2013}{2011}+...+\frac{2013}{2}+\frac{2013}{2013}$

$A=2013\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2013}\right)=2013.B=>\frac{A}{B}=2013$

Bài 6: Tính tỉ số $\frac{A}{B}$ biết : $A=\frac{1}{99}+\frac{2}{98}+\frac{3}{97}+...+\frac{99}{1}$ và $B=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{100}$

HD:

Ta có : $A=\left(\frac{1}{99}+1\right)+\left(\frac{2}{98}+1\right)+...+\left(\frac{98}{2}+1\right)+1=\frac{100}{99}+\frac{100}{98}+...+\frac{100}{2}+\frac{100}{100}$

$A=100\left(\frac{1}{99}+\frac{1}{98}+...+\frac{1}{2}+\frac{1}{100}\right)=100.B=>\frac{A}{B}=100$

Bài 7: Cho $A=\frac{2013}{2}+\frac{2013}{3}+\frac{2013}{4}+...+\frac{2013}{2013}$ và $B=\frac{2013}{1}+\frac{2012}{2}+\frac{2011}{3}+...+\frac{1}{2013}$, tính A/B

Bài 8: Thực hiện phép tính: $\frac{1+\frac{1}{3}+\frac{1}{5}+...+\frac{1}{97}+\frac{1}{99}}{\frac{1}{1.99}+\frac{1}{3.97}+...+\frac{1}{97.3}+\frac{1}{99.1}}$

HD:

Ta có : $TS=\left(1+\frac{1}{99}\right)+\left(\frac{1}{3}+\frac{1}{97}\right)+...+\left(\frac{1}{49}+\frac{1}{51}\right)=100\left(\frac{1}{1.99}+\frac{1}{3.97}+...+\frac{1}{49.51}\right)$

$MS=\left(\frac{1}{1.99}+\frac{1}{99.1}\right)+\left(\frac{1}{3.97}+\frac{1}{97.3}\right)+...+\left(\frac{1}{49.51}+\frac{1}{51.49}\right)=2\left(\frac{1}{1.99}+\frac{1}{3.97}+...+\frac{1}{49.51}\right)$

Khi đó : $\frac{TS}{MS}=\frac{100}{2}=50$

Bài 9: Thực hiện phép tính: $\frac{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{998}+\frac{1}{1000}}{\frac{1}{2.1000}+\frac{1}{4.998}+...+\frac{1}{998.4}+\frac{1}{1000.2}}$

HD:

Ta có : $TS=\left(\frac{1}{2}+\frac{1}{1000}\right)+\left(\frac{1}{4}+\frac{1}{998}\right)+...+\left(\frac{1}{500}+\frac{1}{502}\right)=1002\left(\frac{1}{2.1000}+\frac{1}{4.998}+...+\frac{1}{500.502}\right)$

$MS=\left(\frac{1}{2.1000}+\frac{1}{1000.2}\right)+\left(\frac{1}{4.998}+\frac{1}{998.4}\right)+...+\left(\frac{1}{500.502}+\frac{1}{502.500}\right)$

$MS=2\left(\frac{1}{2.1000}+\frac{1}{4.998}+...+\frac{1}{500.502}\right)$

Khi đó : $\frac{TS}{MS}=\frac{1002}{2}=501$

Bài 10: Tính tỉ số $\frac{A}{B}$ biết: $A=1+\frac{1}{3}+\frac{1}{5}+...+\frac{1}{999}$ và $B=\frac{1}{1.999}+\frac{1}{3.997}+\frac{1}{5.1995}+...+\frac{1}{999.1}$

HD:

Ta có : $A=\left(1+\frac{1}{999}\right)+\left(\frac{1}{3}+\frac{1}{997}\right)+...+\left(\frac{1}{499}+\frac{1}{501}\right)=\frac{1000}{999.1}+\frac{1000}{3.997}+...+\frac{1000}{499.501}$

$=1000\left(\frac{1}{999.1}+\frac{1}{3.997}+..+\frac{1}{499.501}\right)$

$$B=\left(\frac{1}{1.999}+\frac{1}{999.1}\right)+\left(\frac{1}{3.997}+\frac{1}{997.3}\right)+...+\left(\frac{1}{499.501}+\frac{1}{501.499}\right)=\frac{2}{1.999}+\frac{2}{3.997}+...+\frac{2}{499.501}$$

$$=2\left(\frac{1}{1.999}+\frac{1}{3.997}+...+\frac{1}{499.501}\right),\text{ Khi đó : }\frac{A}{B}=\frac{1000}{2}=500$$

Bài 11: Thực hiện phép tính: $A=\dfrac{\frac{1}{51}+\frac{1}{52}+\frac{1}{53}+...+\frac{1}{100}}{\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+...+\frac{1}{99.100}}$

HD:

Ta có : $MS=\frac{1}{1.2}+\frac{1}{3.4}+...+\frac{1}{99.100}=\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{99}-\frac{1}{100}$

$$=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{99}+\frac{1}{100}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{100}\right)$$

$$=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{100}\right)-\left(\frac{1}{1}+\frac{1}{2}+...+\frac{1}{50}\right)=\frac{1}{51}+\frac{1}{52}+\frac{1}{53}+...+\frac{1}{100}=TS$$

Khi đó : $A=\dfrac{TS}{MS}=1$

Bài 12: Tính tỉ số $\dfrac{A}{B}$ biết: $A=\dfrac{2012}{51}+\dfrac{2012}{52}+\dfrac{2012}{53}+...+\dfrac{2012}{100}$ và $B=\dfrac{1}{1.2}+\dfrac{1}{3.4}+\dfrac{1}{5.6}+...+\dfrac{1}{99.100}$

HD:

Ta có : $A=2012\left(\frac{1}{51}+\frac{1}{52}+\frac{1}{53}+...+\frac{1}{100}\right)$

$$B=\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{99}-\frac{1}{100}=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{99}+\frac{1}{100}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{100}\right)$$

$$B=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{100}\right)-\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{50}\right)=\frac{1}{51}+\frac{1}{52}+\frac{1}{53}+...+\frac{1}{100}$$

Khi đó : $\dfrac{A}{B}=\dfrac{2012}{1}=2012$

Bài 13: Tính tỉ số $\dfrac{A}{B}$ biết: $A=\dfrac{1}{1.2}+\dfrac{1}{3.4}+\dfrac{1}{5.6}+...+\dfrac{1}{199.200}$

và $B=\dfrac{1}{101.200}+\dfrac{1}{102.199}+...+\dfrac{1}{200.101}$

HD:

$$A=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{199}-\frac{1}{200}\right)=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{200}\right)-2\left(\frac{1}{2}+\frac{1}{4}+...+\frac{1}{200}\right)$$

$$A=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{200}\right)-\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{100}\right)=\frac{1}{101}+\frac{1}{102}+...+\frac{1}{200}$$

$$A=\left(\frac{1}{101}+\frac{1}{200}\right)+\left(\frac{1}{102}+\frac{1}{199}\right)+...+\left(\frac{1}{150}+\frac{1}{151}\right)=\frac{301}{101.200}+\frac{301}{102.199}+...+\frac{301}{150.151}$$

Và $B=\left(\frac{1}{101.200}+\frac{1}{200.101}\right)+\left(\frac{1}{102.199}+\frac{1}{199.102}\right)+...+\left(\frac{1}{150.151}+\frac{1}{151.150}\right)$

$$B=\frac{2}{101.200}+\frac{2}{102.199}+...+\frac{2}{150.151}$$

Khi đó : $\dfrac{A}{B}=\dfrac{301}{2}$

Bài 14: Tính giá trị $\frac{A}{B}$ biết: $A=\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+...+\frac{1}{101.102}$ và

$B=\frac{1}{52.102}+\frac{1}{53.101}+\frac{1}{54.100}+...+\frac{1}{102.52}+\frac{2}{77.154}$

HD:

Ta có : $A=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\frac{1}{101}-\frac{1}{102}=\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{101}-\frac{1}{102}\right)$

$A=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{101}+\frac{1}{102}\right)-2\left(\frac{1}{2}+\frac{1}{4}+...+\frac{1}{102}\right)$

$A=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{102}\right)-\left(\frac{1}{1}+\frac{1}{2}+...+\frac{1}{51}\right)=\frac{1}{52}+\frac{1}{53}+...+\frac{1}{101}+\frac{1}{102}$

$A=\left(\frac{1}{52}+\frac{1}{102}\right)+\left(\frac{1}{53}+\frac{1}{101}\right)+...+\left(\frac{1}{76}+\frac{1}{78}\right)+\frac{1}{77}=\frac{154}{52.102}+\frac{154}{53.101}+...+\frac{154}{76.78}+\frac{154}{77.154}$

và $B=\left(\frac{1}{52.102}+\frac{1}{102.52}\right)+\left(\frac{1}{53.101}+\frac{1}{101.53}\right)+...+\left(\frac{1}{76.78}+\frac{1}{78.76}\right)+\frac{2}{77.154}$

$B=\frac{2}{52.102}+\frac{2}{53.101}+...+\frac{2}{76.78}+\frac{2}{77.154}=>\frac{A}{B}=\frac{154}{2}=77$

Bài 15: Cho $A=\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+...+\frac{1}{101.102}$;

$B=\frac{1}{52.102}+\frac{1}{53.101}+\frac{1}{54.100}+...+\frac{1}{101.53}+\frac{1}{102.52}$

Chứng tỏ rằng $\frac{A}{B}$ là số nguyên.

Bài 16: CMR: $\left(1+\frac{1}{3}+\frac{1}{5}+...+\frac{1}{99}\right)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{100}\right)=\frac{1}{51}+\frac{1}{52}+...+\frac{1}{100}$

HD:

Ta có : $VT=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{99}+\frac{1}{100}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+...+\frac{1}{100}\right)$

$VT=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{100}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{50}\right)=\frac{1}{51}+\frac{1}{52}+...+\frac{1}{100}=VP$

Bài 17: Cho $S=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{2011}-\frac{1}{2012}+\frac{1}{2013}$ và $P=\frac{1}{1007}+\frac{1}{1008}+...+\frac{1}{2012}+\frac{1}{2013}$.

Tính $(S-P)^{2013}$

HD:

Ta có : $S=\left(1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{2013}\right)-2\left(\frac{1}{2}+\frac{1}{4}+...+\frac{1}{2012}\right)$

$S=\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2013}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{1006}\right)=\frac{1}{1007}+\frac{1}{1008}+...+\frac{1}{2013}=P$

Khi đó : $(S-P)^{2013}=0^{2013}=0$

Bài 18: Chứng minh rằng: $100-\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{100}\right)=\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+...+\frac{99}{100}$

HD:

Ta có : $VT=(1-1)+\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{3}\right)+...+\left(1-\frac{1}{100}\right)=\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+...+\frac{99}{100}=VP$ (đpcm)

Bài 19: Tính tỉ số $\frac{A}{B}$ biết : $A=92-\frac{1}{9}-\frac{2}{10}-\frac{3}{11}-...-\frac{92}{100}$ và $B=\frac{1}{45}+\frac{1}{50}+\frac{1}{55}+...+\frac{1}{500}$

HD:

Ta có : $A=\left(1-\frac{1}{9}\right)+\left(1-\frac{2}{10}\right)+\left(1-\frac{3}{11}\right)+...+\left(1-\frac{92}{100}\right)=\frac{8}{9}+\frac{8}{10}+...+\frac{8}{100}=8\left(\frac{1}{9}+\frac{1}{10}+...+\frac{1}{100}\right)$

$B=\frac{1}{5}\left(\frac{1}{9}+\frac{1}{10}+...+\frac{1}{100}\right)$. Khi đó : $\frac{A}{B}=\frac{8}{\frac{1}{5}}=40$

Bài 20: Cho $A=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...-\frac{1}{2016}+\frac{1}{2017}-\frac{1}{2018}$ và $B=\frac{1}{1010}+\frac{1}{1011}+...+\frac{1}{2016}+\frac{1}{2017}+\frac{1}{2018}$

Tính $\left(A^{2017}-B^{2017}\right)^{2018}$

DẠNG 12: TÍNH GIÁ TRỊ BIỂU THỨC

Bài 1: Cho abc=2015, Tính $A=\frac{2015a}{ab+2015a+2015}+\frac{b}{bc+b+2015}+\frac{c}{ac+c+1}$

HD :

$$A=\frac{a^2bc}{ab+a^2bc+abc}+\frac{b}{bc+b+abc}+\frac{c}{ac+c+1}$$

$$=\frac{a^2bc}{ab(1+ac+c)}+\frac{b}{b(c+1+ac)}+\frac{c}{ac+c+1}=\frac{ac+c+1}{ac+c+1}=1$$

Bài 2: Cho abc=2, Tính $B=\frac{a}{ab+a+2}+\frac{b}{bc+b+1}+\frac{2c}{ac+2c+2}$

HD :

$$B=\frac{a}{ab+a+abc}+\frac{b}{bc+b+1}+\frac{abc^2}{ac+abc^2+abc}=\frac{a}{a(b+1+bc)}+\frac{b}{bc+b+1}+\frac{abc^2}{ac(1+bc+b)}=1$$

Bài 3: Cho abc=1, Tính $A=\frac{a}{ab+a+1}+\frac{b}{bc+b+1}+\frac{c}{ac+c+1}$

HD :

$$A=\frac{a^2bc}{ab+a^2bc+abc}+\frac{b}{bc+b+abc}+\frac{c}{ac+c+1}=\frac{a^2bc}{ab(1+ac+c)}+\frac{b}{b(c+1+ac)}+\frac{c}{ac+c+1}=1$$

Bài 4: Cho $xyz=1$, Tính giá trị của: $A=\frac{x}{xy+x+1}+\frac{y}{yz+y+1}+\frac{z}{xz+z+1}$

Bài 5: Cho abc= - 2012, Tính $B=\frac{a}{ab+a-2012}+\frac{b}{bc+b+1}-\frac{2012c}{ac-2012c-2012}$

HD :

$$B=\frac{a}{ab+a+abc}+\frac{b}{bc+b+1}+\frac{abc^2}{ac+abc^2+abc}=\frac{a}{a(b+1+bc)}+\frac{b}{bc+b+1}+\frac{abc^2}{ac(1+bc+b)}=1$$

Bài 6: Chứng minh rằng nếu xyz=1 thì $\frac{1}{1+x+xy}+\frac{1}{1+y+yz}+\frac{1}{1+z+zx}=1$

HD :

$$VT=\frac{xyz}{xyz+x^2yz+xy}+\frac{xyz}{xyz+y+yz}+\frac{1}{1+z+zx}=\frac{xyz}{xy(z+xz+1)}+\frac{xyz}{y(xz+1+z)}+\frac{1}{1+z+zx}=1=VP$$

Bài 7: Cho xyz=2010, CMR: $\frac{2010x}{xy+2010x+2010}+\frac{y}{yz+y+2010}+\frac{z}{xz+z+1}=1$

HD :

$$VT=\frac{x^2yz}{xy+x^2yz+xyz}+\frac{y}{yz+y+xyz}+\frac{z}{xz+z+1}=1$$

Bài 8: Tính giá trị của biểu thức : $A=13a+19b+4a-2b$ với a+b=100

HD:

Ta có : $A=(13a+4a)+(19b-2b)=17a+17b=17(a+b)=17.100=1700$

Bài 9: Tính giá trị của biểu thức: $5x^2+6x-2$ khi $|x-1|=2$

HD:

Ta có : Khi $|x-1|=2 \Rightarrow \left[\begin{array}{l} x-1=2 \\ x-2=-2 \end{array}\right. \Rightarrow \left[\begin{array}{l} x=3 \\ x=0 \end{array}\right.$

Khi $x=3 \Rightarrow A=5x^2+6x-2=5.9+6.3-2=61$. Khi $x=0 \Rightarrow A=5x^2+6x-2=-2$

Bài 10: Tính giá trị của biểu thức: $2x^5-5y^3+4$, biết $(x-1)^{20}+(y+2)^{30}=0$

HD:

Ta có : Vì $\begin{cases}(x-1)^{20}\ge 0\\(y+2)^{30}\ge 0\end{cases}\Rightarrow (x-1)^{20}+(y+2)^{30}=0\Rightarrow\begin{cases}x-1=0\\y+2=0\end{cases}\Rightarrow\begin{cases}x=1\\y=-2\end{cases}$, Thay vào ta được :

$A=2.1^5-5.(-2)^3+4=2+40+4=46$

Bài 11: Cho a, b,c khác 0 và đôi 1 khác nhau thỏa mãn : $a^2(b+c)=b^2(a+c)=2013$, Tính $A=c^2(a+b)$

HD:

Ta có :

$a^2(b+c)=b^2(a+c)=2013\Rightarrow a^2b+a^2c-b^2a-b^2c=0\Leftrightarrow ab(a-b)+c(a-b)(a+v)=0$

$(a-b)(ab+bc+ca)=0\Rightarrow ab+bc+ca=0$ vì $a\ne b$

Khi đó : $(ab+bc+ca)b=0\Rightarrow b^2(a+c)=-abc\Rightarrow -abc=2013$

tương tự : $(ab+bc+ca)c=0\Rightarrow c^2(a+b)=-abc=2013$

Bài 12: Cho $A=\dfrac{1,11+0,19-1,3.2}{2,06+0,54}-\left(\dfrac{1}{2}+\dfrac{1}{3}\right):2$ và $B=\left(5\dfrac{7}{8}-2\dfrac{1}{4}-0,5\right):2\dfrac{23}{26}$

a, Rút gọn A và B

b, Tìm x nguyên sao cho: $A<x<B$

HD:

a, Ta có :

$A=\dfrac{-1}{2}-\left(\dfrac{5}{6}\right):2=\dfrac{-1}{2}-\dfrac{5}{12}=\dfrac{-11}{12}$, Và $B=\dfrac{25}{8}:\dfrac{75}{26}=\dfrac{13}{12}$

b, Ta có :

$A<x<B\Rightarrow\dfrac{-11}{12}<x<\dfrac{13}{12}\Rightarrow\dfrac{-11}{12}<\dfrac{12x}{12}<\dfrac{13}{12}\Rightarrow -11<12x<13\Rightarrow\left[\begin{array}{l}12x=0\\12x=12\end{array}\right.$

Bài 13: Cho $P=|2a-1|-(a-5)$

a, Rút gọn P

b, Có giá trị nào của a để P=4 không?

HD:

Ta có :

a, $P=\left[\begin{array}{l}2a-1-a+5,vs\left(a\ge\dfrac{1}{2}\right)\\1-2a-a+5,\left(a<\dfrac{1}{2}\right)\end{array}\right.=\left[\begin{array}{l}a+4\left(a\ge\dfrac{1}{2}\right)\\6-3a\left(a<\dfrac{1}{2}\right)\end{array}\right.$ b , Để $P=4\Rightarrow\left[\begin{array}{l}a+4=4\left(a\ge\dfrac{1}{2}\right)\\6-3a=4\left(a<\dfrac{1}{2}\right)\end{array}\right.\Rightarrow\left[\begin{array}{l}a=0(l)\\a=\dfrac{2}{3}(l)\end{array}\right.$

Vậy không có giá trị nào của a đề P =4